Q Skills Review

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Measurement 1: Units of Measurement

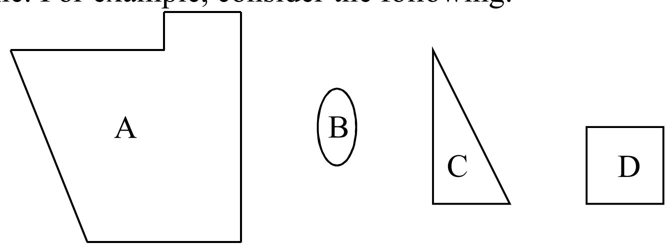
In life we often want to quantify an attribute so that we can then communicate with others or make comparisons. For example, how tall are you, how far is it to Calgary, which room is larger, which jug holds more water, which rugby team is heavier? Originally people used whatever was convenient to measure quantities, such as the length of a step or the width of a hand, the amount held in a cup or a spoon. However, my hand may be smaller than yours, or my cup may be larger. For trading purposes people wanted to know that the measurements used by different people were actually the same size, and so standard units of measure were adopted, at first locally, and then in wider circles as trade spread. Different countries used different standard measurements, and over the centuries there has been a gradual process of redefining units of measure, or adoption of new units, to help communication so that now almost all countries use the International System of Units (the metric system). Metrication began in France in the 1790s and, although most countries of the world have adopted the metric system, some, including Canada, are changing gradually, with traditional units still being used alongside metric for some purposes. Only the United States, Liberia, and Myanmar have not adopted it as their primary or sole system of measurement (although Myanmar uses metric units in daily life). The United States was actually one of the original seventeen signatory nations to the 'Convention du Mètre' in 1875, and the 'Metric Conversion Act' of 1975 stated that "it is therefore the declared policy of the United States to designate the metric system of measurement as the preferred system of weights and measures for United States trade and commerce." The transition to the metric system has still not fully taken hold in the USA, although it is the system used for most scientific purposes.

Why is it important which units we use? Here is an example of the problems which can arise if we do not use standard units and if we do not state clearly which units we have used: In 1998 NASA sent a spacecraft to orbit Mars. NASA used metric units, but one major contractor provided numerical data which assumed traditional units. The end result was that the spacecraft descended too close to the planet and burned up in the atmosphere before it could fulfil its mission. The cost of the failed mission: over 300 million dollars!

Measurement can be confusing not only because of the mixture of metric and traditional units in common use, but even understanding what quantity we are measuring may not be simple. Length is a concept grasped by most adults, but are we sure exactly what we are measuring when it comes to area and volume? Is there a difference between volume and capacity? We may have an instinct for time and temperature, but the difference between mass and weight can cause some confusion. What about other quantities, such as energy? Perhaps we have a sense of what energy is, but what are the units? Most people have heard of calories (a unit of energy), but did you know that 1 Calories = 1000 calories? The metric unit of energy, the Joule, is less well recognised, and the unit used for energy on our electricity bill is the kilowatt-hour (kWh). For this unit we have a combination of Watts and hours (multiplied) and we have a prefix 'kilo' (meaning thousand). Another common situation where we combine units is when we are describing rates. For example, speed (the rate at which an object is moving) can be measured in kilometres per hour (km/h or kmh^{-1}), and we are dividing rather than multiplying. We must pay attention to the details: **C**al or **c**al, ... **h** or ... /**h**, or ... $\mathbf{h^{-1}}$ (which is a different notation for /h).

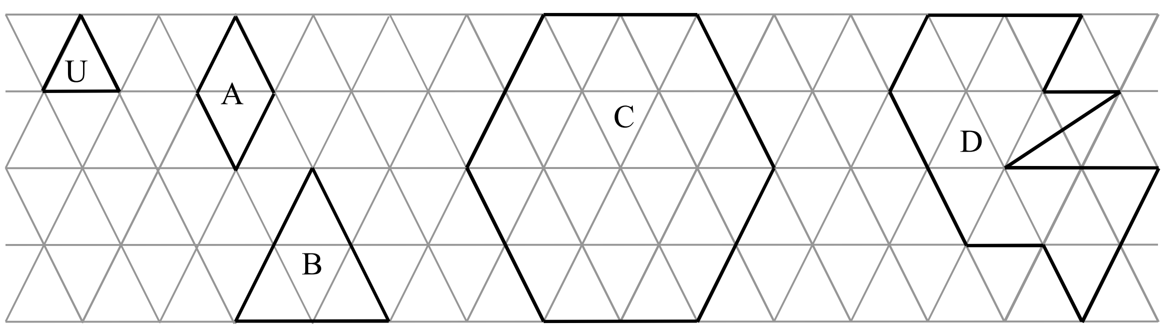
Area and Volume

Most adults have an intuitive grasp of the concept of length as the distance between two points along a straight line between those points. To measure a distance not in a straight line, we can add several shorter straight line distances together. What exactly is area? It can be described as an amount of surface within a closed boundary. We can compare the areas of different regions, saying that one is larger than another, or that they are the same, even if the actual shapes of the regions are not the same. For example, consider the following:



We can easily see that shape A covers more surface than shape B, so we say that the area of A is greater than the area of B. Shapes C and D cover an amount of surface smaller than that covered by shape A, and larger than that covered by shape B, but it is not immediately obvious how they compare to each other.

As with all measurements, when we want to describe the size of an area, it is helpful to compare it to a common unit of measurement. We can choose a simple shape and define the area of that shape to be the unit. The unit does not have to be a square - consider the following example:



We can describe the amount of surface covered by one triangle (see shape U) as the unit, i.e. one 'triangular unit'. The area of shape A can then be described as 2 triangular units. Shape B has an area of 4 triangular units, shape C has an area of 24 triangular units, and shape D has an area of 14 triangular units.

Almost all standard units of area are defined as the area of a square whose side length is a standard unit of length, hence we get square metres, square feet, and so on. One exception to this is the acre – the definition of an acre is discussed in the section on traditional units.

If area is 'an amount of surface', then we can describe volume as an amount of space. Strictly speaking, the volume of an object is the amount of space which the material of the object occupies. If it is not solid, then the amount of space inside the object is called its capacity (how much it holds). However, this distinction is often ignored, and the word 'volume' is commonly used for amount of space contained within the boundaries of the shape, whether it is solid or not.

As with measuring area, we need to define a unit of volume in order to quantify the volume of a shape. Just as the standard shape for quantifying areas is the square, so the standard shape for quantifying volumes is the cube. In general, the unit is taken to be a cube whose side length is a standard unit of length, giving cubic metres, cubic feet, etc. A litre is a metric unit defined as the volume of a cube with side length 10 cm, and so 1 litre = 10 cm $\times$ 10 cm $\times$ 10 cm = 1000 cm^3. The litre is often used as the unit for capacity, or for the volume of liquids and gases.

Mass and Weight

The mass of an object is the amount of matter (stuff) in the object. The weight of the object is the force exerted by that object due to gravity. Since gravity is approximately the same everywhere on the earth's surface then the measurement of weight indicates the mass of an object and the mass of an object determines its weight. This has led to 'mass' and 'weight' being used interchangeably in everyday language. If you were to take an object to the moon, where gravity is much weaker than on earth, the weight of the object would be much less, even though the mount of matter (stuff) would remain the same. Technically, the kilogram is a unit of mass and the pound is a unit of weight, but this distinction is rarely made outside physics and related fields. [Many traditional systems of have units for weight, but not for mass. However, in the Imperial system the 'slug' is a unit of mass!]

Rates

A rate involves two different quantities, and expresses the amount of the first quantity for each unit of the second. For example, when we measure a distance, we have one quantity; but if we also measure the time taken for an object to travel a distance, then we have two quantities – distance and time. If we give the distance the object travels in one unit of time then we are describing a rate (the speed). Similarly, we can count a number of heart beats (one quantity), and we can measure the amount of time for which we were counting (a second quantity). When we give the number of heart beats counted during one unit of time we are giving the rate at which the heart is beating. The units for rates are formed by linking the units used to measure each of the two quantities with the word 'per', e.g. kilometres per hour, beats per minute, etc. In fact, the units of the rate can tell us the quantities involved. For example, a concentration of 3.2 grams per litre means that the two quantities are measured in grams and litres, so we have 'mass' and 'volume'. 3.2 grams per litre means that each litre of the solution contains 3.2 grams of the substance.

In some situations it may not be immediately obvious whether we have a single quantity or a rate. For example, GDP (gross domestic product) is the total value of all goods and services produced by a country. We may not always find it useful to compare the GDP of a country with a large population and that of a country with a small population and so the GDP per capita may be used, that is, the GDP divided by the total population, giving the average amount for each person in the country. GDP is frequently measured in US dollars to allow comparisons between countries, but GDP per capita is also often given in US dollars, rather than writing US$/person. Care must be taken when reading or communicating this information to be sure which measure is intended.

The Metric System

Historically each country developed its own system of measurements with units that were meaningful in the situations involved. However, this has led to units being defined differently in different countries, such as the US gallon and the British gallon. Also, the relationships between units can be difficult to work with and remember, for example 12 inches in a foot, 5280 feet in a mile. The metric system is a decimal system (based on powers of ten to relate units, e.g. 1 cm = 10 mm, 1 m = 100 cm) and has been developed into an international system of units (SI units), adopted by the majority of the world (although many countries still use some traditional units in everyday life).

Here are a few comments on the metric system of units:

- The metric system formally defines base units, and then units which are derived from these base units. Within each discipline you will become familiar with a variety of other units and learn their definitions and relationships. For now we will focus on some of the commonly used units with which you are already familiar.

Attribute	**Unit Name**	**Unit Symbol**
length	metre[1]	m
area	square metre	m^2
volume	cubic metre	m^3
capacity	litre[1]	l or L
mass	gram[2], tonne	g, t
time	second, hour	s, h
temperature	degree Celsius	ºC

[1] In the US the spelling is meter and liter, but met<u>re</u> and lit<u>re</u> are the official spellings in Canada and all other countries.

[2] The kilogram is used in the definition of derived units, but the naming system uses the gram as if it were the base unit.

- By using prefixes (such as 'milli-' or 'kilo-') we can describe smaller or larger quantities.
- The tonne is a special name for 1000 kilograms and does not follow the usual system of naming.
- The units for area (or volume) are derived from the units of length by considering a square (or cube) with a unit side length. The unit name is 'square …', e.g. square metre, or 'cubic …', e.g. cubic metre (and <u>not</u> 'metre squared' or 'metre cubed', as is often mistakenly said because of the order of the symbols).
- Unit symbols are generally small letters (m, g, t), unless the name is derived from the name of a person (ºC). The exception to this is the symbol for litre, which was originally l, but the use of L is now also acceptable in order to avoid confusion with the number 1 when typed.
- The connection between the units of capacity and those for volume is that 1 litre = $1000cm^3$. This can be expressed as 1 ml = $1cm^3$. You will need to know this connection, so choose one of these forms to memorise.

Metric Prefixes

Prefixes can be used with any metric unit to indicate multiples (or fractions) of that unit. Each time the quantity is multiplied (or divided) by one thousand there is a new prefix. The same prefixes are used for all units, for example, 1000 metres is 1 kilometre (km) and 1000 grams is 1 kilogram (kg), and 1000 watts is 1 kilowatt (kW). Prefixes up to 10^{24} and down to 10^{-24} have been named, and as technology progresses, more of these prefixes have become familiar to non-scientists.

Multiple	Power of 10	Prefix	Prefix symbol
1 000 000 000 000	10^{12}	tera-	T
1 000 000 000	10^{9}	giga-	G
1 000 000	10^{6}	mega-	M
1 000	10^{3}	kilo-	k
1	10^{0}	no prefix	unit symbol
$\frac{1}{1000} = 0.001$	10^{-3}	milli-	m
$\frac{1}{1000000} = 0.000001$	10^{-6}	micro-	μ
$\frac{1}{1000000000} = 0.000000001$	10^{-9}	nano-	n
$\frac{1}{1000000000000} = 0.000000000001$	10^{-12}	pico-	p

In general, prefixes indicating large multiples use upper case letters and those indicating a fraction of a unit use lower case letters. There are two exceptions to this: ‘kilo-’ uses a lower case ‘k’, and the Greek letter μ (pronounced ‘mew’) is used for ‘micro-’, since ‘M’ and ‘m’ have already been used for ‘mega-’ and ‘milli-’.

For convenience, sometimes prefixes for multiples between $\frac{1}{1000}$ and 1000 are used (all lower case). The most common of these is centi-, as in centimetre (cm) for measuring small lengths.

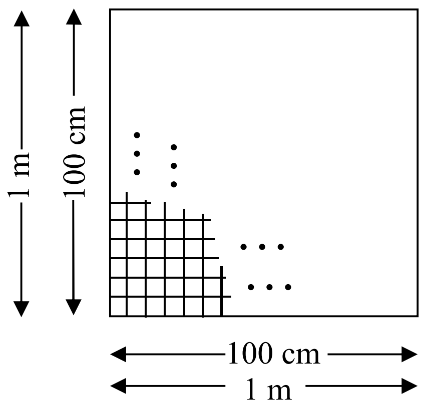
Multiple	Prefix	Symbol
100	hecto	h
10	deca	da
1	no prefix	unit symbol
$\frac{1}{10} = 0.1$	deci-	d
$\frac{1}{100} = 0.01$	centi-	c

Changing Metric Prefixes

Converting between metric units with different prefixes is easy, since we must simply multiply or divide by powers of ten (e.g. 1000 or 100, etc.). For example, change 70 mg to grams: since milli- means one-thousandth we have 1 mg = $\frac{1}{1000}$ g, and so 70 mg = $\frac{70}{1000}$ g = 0.07 g. Always use common sense to check that you have converted the ‘correct way’: since 1000 milligrams = 1 gram, it makes sense that 70 mg is less than one gram.

When converting between units of area or volume you must keep in mind how the unit is defined. Drawing a picture of the unit can make this easier. For example, 1 m^2 = ___?___ cm^2.

First put the question into words: one square metre covers the same area as how many square centimetres? Now draw a diagram to illustrate this.



We see that 100 small squares fit along the edge of the large square, and so there are 100 × 100 small squares in the large square. Hence the area covered by one square metre is the same as the area covered by 100 × 100 = 10,000 square centimetres. In symbols, $1\ m^2 = 10{,}000\ cm^2$

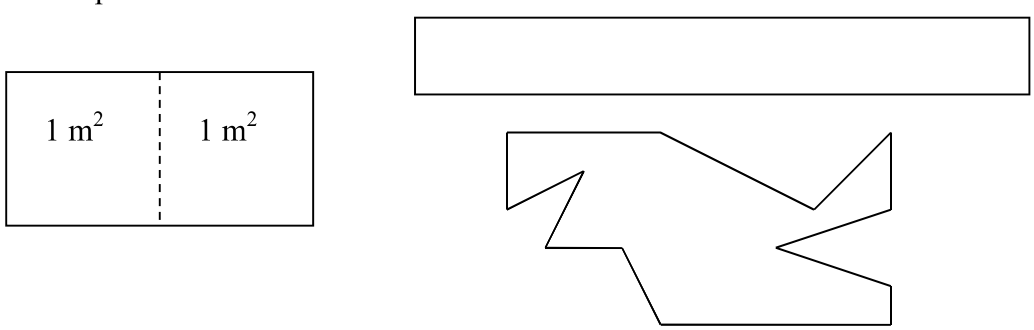
Draw a diagram to illustrate the following:

$$1\ m^3 = \underline{\quad ? \quad}\ cm^3$$

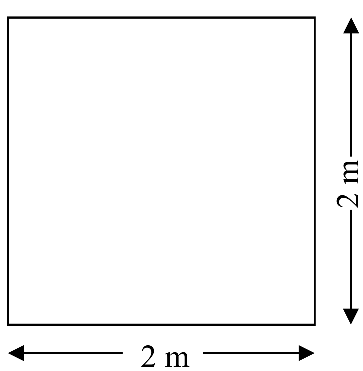
[You should find that there are a million cubic centimetres in a cubic metre.]

There is a potential confusion if we do not pay attention to the correct names of the units for area and volume. Consider this: what is 2 square metres, and what is 2 metres squared?

2 square metres is a measurement of the amount of surface covered. The following drawings represent shapes that all have an area of 2 square metres, but we cannot know what the shape is if all we know is its area.



2 metres squared, on the other hand, describes a square which has been formed by taking a length of 2 metres along each side:



What is the area of this square?

The length of the square is 2 m and so 2 unit squares would fit along the edge. [See dotted lines.]
We can fit four unit squares into this shape, and so the area of a shape which is 2 metres squared is 4 square metres.

This is why it is very important to read m^2 as 'square metres' rather than metres squared!

In symbols: 2 square metres is $2\ m^2$, whereas 2 metres squared is $(2\ m)^2$, and $(2\ m)^2$ means $(2\ m) \times (2\ m) = 4\ m^2$.

Mathematicians usually do not write the units in the middle of calculations, unless converting from one unit to another. For example, suppose we have a square with side-length 30 cm, and want to calculate the area and perimeter. A mathematician might write:

Area = 30 × 30 = 900 cm^2 and Perimeter = 4 × 30 = 120 cm = 1.2 m

However, a scientist might write:

Area = 30 cm × 30 cm = 900 cm^2 and Perimeter = 4 × 30 cm = 120 cm = 1.2 m

Whenever the value of a measurement is given, the unit is a necessary part of this value, and so it is common in scientific calculations to write the units with the number at all stages. Look out for this and do not confuse the letters used for units with the letters used algebraically. To help with this, it is conventional to use italics for variables and unknowns and regular font for units (but of course this is not easy to show in hand writing). For example, 2*n* m indicates a length measured in metres with numerical value twice the value of *n*, whereas 2 nm indicates a length measured in nanometres with numerical value 2.

How big are these units?

It is useful to consider some everyday objects which will remind you of the size of the various units. You can add your own ideas to the table.

Attribute	**Unit**	**Everyday example**
length	mm	thickness of a credit card
	cm	length of nail on smallest finger 10 cm : width of a hand
	m	height of a kitchen counter 100 m : length of a football field
	km	distance walked in 10 – 20 mins distance across Canada ~ 4000 to 5000 km
area	cm^2	area of a small postage stamp sheet of letter size paper ~ 600 cm^2
	m^2	area of a large coffee table top area of a football field ~ 5000 to 8000 m^2
	km^2	area of a city park or small golf course area of Canada, USA or China ~ 9 to 10 million km^2
volume/capacity	ml or cm^3	volume of a sugar cube volume of drinks can 355 ml
	l (or L)	volume of a carton of milk water in a bathtub ~ 200 litres
	m^3	volume of a floor to ceiling kitchen cupboard olympic swimming pool 2500 m^3 (2.5 million litres)
weight	mg	weight of a grain of sand weight of 1 m of sewing thread ~ 20 mg
	g	weight of a paper clip weight of a baseball ~ 150 g
	kg	weight of a 1 litre bottle of water weight of a person 60 to 80 kg
	t	weight of a small car max weight of loaded road trucks 50 to 60 tonnes

Practice Questions Set 1

[Answers to these questions can be found at the end.]

1) (a) 1.6 m = ______ cm (b) 1.6 cm = ______ m (c) 0.45 kg = ______ g
   (d) 0.25 L = ______ mL (e) 3.4 Mt = ______ t (f) 62.5 nm = ______ mm
   (g) 1 h = ______ s (h) 7 days = ______ min (i) 500 min = ____ h ____ min

2) A rectangle is 1.3 m long and 60 cm wide. By expressing the width in metres, find the area of the rectangle in square metres. Then give the length in centimetres and use this to find the area in square centimetres. Finally, show that the two values are actually the same.

3) Give the most appropriate metric units for the following quantities:
   (a) The height of a tall building.
   (b) The mass of a house key.
   (c) The land area of BC.
   (d) The capacity of a mug.
   (e) The volume of a ball.
   (f) The area of a carpet.
   (g) The mass of a passenger train carriage.
   (h) The thickness of a sheet of paper.
   (i) The area of a kitchen cupboard door.
   (j) The mass of full backpack.
   (k) The capacity of a children's paddling pool.
   (l) The volume of concrete in driveway.

4) (a) Express 720 cm^3 in litres.
   (b) Convert 0.0032 m^3 to cubic centimetres.
   (c) Express 350000 cm^3 in both cubic metres and litres.

5) Explain the difference in meaning between the phrases '4 metres squared' and '4 square metres'. Include in your explanation how you would write each of them using symbols.

6) In each of the following, a number has been calculated using the given information. Complete the final sentence of each by giving the units.
   (a) I completed the 10 km fun run in exactly 2 hours. I averaged 5 ______.
   (b) After working 4 hours, Tim is paid $88. He is paid 22 ______.
   (c) A summer worker planted 6000 trees and was paid $1800. She was paid 0.3 ______.
   (d) A hockey player was on the ice for 1080 minutes this season, in which he played 45 games. His stats listed "time on ice" as 24 ______.

7) For international financial comparisons it is common for the all values to be given in US dollars. When looking at information about public debt, some sources say that it is about the same in Canada ($34,900) and the USA ($36,600), whereas other sources say that the USA's public debt is almost ten times that of Canada ($11,600 billion and $1,200 billion respectively). Explain why these seemingly very different conclusions are not inconsistent. How could the units given with the numbers have helped readers understand the information?

Traditional Units

Canada and many other countries continue to use non-metric units in some situations. In the supermarket you will still see prices of fresh produce given in 'cents per pound', and people often give their height in feet and inches. For the commonly used traditional units it is useful to know the main relationships, and also to know an approximate metric equivalent.

A word of caution: the use of traditional units is more common in the USA than in most countries, and this is often referred to as the 'English system' of units. This name is confusing, since this system is not quite the same as that used in England! The units used in the USA are officially called 'US Customary Units', whereas in Britain, and some other countries with historical ties to Britain, the 'Imperial System of Units' is still in partial use. Many units are the same in both systems, but there are some differences in definitions (even though the names are the same). For example, the US fluid ounce is slightly larger than the Imperial fluid ounce, and there are 16 fluid ounces in a pint in the US, but 20 fluid ounces in an Imperial pint. This leads to the gallon being quite different in the US and Britain. Also, the US ton is smaller than the Imperial ton. According to a page on the Government of Canada website, the Canadian units of measurement have the fluid ounce, pint and gallon the same as the Imperial units, but the ton is the same as the US ton! If you are interested in the measurement systems used in different countries, or their historic development, there are several interesting discussions on Wikipedia.

The following table gives a summary of some traditional units still used in Canada, along with their relationships and the approximate metric equivalents.

Units of Length	**Symbol**	**Relationship**	**Metric Equivalence**
inch	in		2.54 cm (exactly)
foot	ft	12 inches	~ 30 cm
yard	yd	3 feet	~ 1 m (bit less)
mile	– no symbol –	5280 feet	~ 1.6 km
Units of Area			
square … (insert any unit of length)	sq (length unit)		
acre	ac	640 acres = 1 sq.mile	1 hectare ≈ 2.5 acres
Units of Volume			
cubic … (insert any unit of length)	cu. (length unit)		
Units of Capacity			
fluid ounce	fl oz		~ 30 ml (bit less)
pint (Canadian)	pt	20 fl oz (Canada)	~ 570 ml
pint (US)		16 fl oz (US)	~ 470 ml
quart	qt	2 pints	
gallon	gal	4 quarts	Canada ~ 4.5 litres US ~ 3.8 litres
Units of Weight			
ounce	oz		~ 28 g
pound	lb	16 ounces	1 kg ≈ 2.2 lbs
ton (short) (Canada, US)	– no symbol –	2000 pounds	~ 900 kg = 0.9 tonne
ton (long) (Imperial)	– no symbol –	2240 pounds	~ 1 tonne

Notice that, just like the metric system, units of area are defined as the area of a square with side length one unit, and units of volume are defined as the volume of a cube with side length one unit. The exception to this is the acre, which is defined as the area of a rectangle with length one furlong and width one chain. The history of this unit is interesting – the word ‘acre’ was also the Saxon word for ‘field’. The furlong was the length of the traditional furrow as ploughed by ox teams, and the amount of land an ox team could plough in a day was called an acre. Furlongs are still used in horse racing (8 furlongs = 1 mile, so 1 furlong = 220 yards), and a chain is the distance between the wickets on a cricket pitch (22 yards).

Practice Questions Set 2

[Answers to these questions can be found at the end.]

1) (a) 8 ft = ______ in (b) 26400 ft = ______ miles (c) 4 gal = ______ pt
   (d) 32 oz = ______ lb (e) 5 ft 7 in = ______ in (f) 52 oz = ___ lb ___ oz

2) (a) 6 sq yd = ______ sq ft (b) 720 sq in = ______ sq ft (c) 135 cu ft = ______ cu yd

3) (a) 12 fl oz ≈ ______ ml (b) 60 kg ≈ ______ lb (c) 55 miles ≈ ______ km
   (d) 420 g ≈ ______ oz (e) 6 ft ≈ ______ m (f) 500 L ≈ ______ gal

Converting Units

We have already made conversions between units by considering the metric prefixes or using known relationships such as 1 foot = 12 inches. For example, the following examples show conversions which are quite simple and do not need any special technique.

Express 4 hour in minutes: 4 hours = (4×60) mins = 240 mins
Express 30 cm in metres: 30 cm = (30 / 100) m = 0.30 m
Express 5 inches in cm: 5 in = (5×2.54) cm = 12.7 cm

It becomes slightly more complicated when we have two or more steps, such as:
Express 6 feet in cm: 6 ft = (6×12) in = 72 in = (72×2.54) cm = 182.88 cm

When we have even more steps, or units which involve more than one dimension, such as rates, it can be quite demanding to think through the steps involved. For example, Olympic sprinters can reach a top speed of around 12 m/s, but to appreciate how fast they are running we may want to convert this to km/h (or mph). We can think this through step by step as follows:

12 m/s means 12 m in 1 s, which is (12×60) m in 1 min = 720 m in 1 min,

which is (720×60) m in 1 hour = 43200 m in 1 hour,

which is (43200/1000) km in 1 hour = 43.2 km/h.

Since 1 mile ≈ 1.6 km, then 43.2 km in an hour ≈ (43.2/1.6) miles in an hour = 27 mph.

We will now discuss a technique which allows us to make these more complicated conversions in a systematic way and in a single calculation. The technique is based on two important ideas:

(i) Relationships between units (conversion ratios) can be expressed as fractions which are equal to one. For example, 1 m = 100 cm, and so $\frac{1\text{ m}}{100\text{ cm}} = \frac{100\text{ cm}}{100\text{ cm}} = 1$.

(ii) When we multiply a value by one then the value is not changed, e.g. 260 cm = 260 cm × 1.

Combining these ideas, and using our knowledge of fractions to simplify the result, we can use multiplication by fractions to convert units.

For example, 260 cm = 260 cm × 1 = $260\text{ cm} \times \frac{1\text{ m}}{100\text{ cm}} = \frac{260\text{ cm}}{100\text{ cm}} \times 1\text{ m} = 2.6 \times 1\text{ m} = 2.6\text{ m}$

In practice, we can save some of the writing by simply 'cancelling' any units which appear in both the numerator and the denominator: $260\text{ cm} = 260\ \cancel{\text{cm}} \times \frac{1\text{ m}}{100\ \cancel{\text{cm}}} = \frac{260\times 1}{100}\text{ m} = 2.6\text{ m}$

[For more about working with fractions, see the Q Skills Review "Fractions".]

Revisiting the example of converting 12 m/s into km/h, we get

$$12\text{ m/s} = \frac{12\ \cancel{\text{m}}}{1\ \cancel{\text{s}}} \times \frac{1\text{ km}}{1000\ \cancel{\text{m}}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\text{ h}} = \frac{(12\times 1\times 60\times 60)\text{ km}}{(1\times 1000\times 1\times 1)\text{ h}} = 4.32\text{ km/h}$$

Reflecting on this example we note the following:

- 12 m/s means 12 m in 1 s, and so it can be written as $\frac{12\text{ m}}{1\text{ s}}$.
- We need to convert the units from m/s to km/h, so we must know the conversion facts relating metres to kilometres, and seconds to hours (perhaps via minutes).
- There is no doubt about which way to write the ratio fraction, since we cancel units in the numerator with units in the denominator. If we write the fractions the 'wrong way up', then the units will not cancel, e.g. if we write $\frac{12\text{ m}}{1\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}$, we cannot cancel the 'm's (and the 'km' is in the denominator, rather than the numerator).

When converting square or cubic units, the conversion ratio for the basic units must be applied two (or three) times, once for each dimension. This is illustrated in the following example.

Convert 15 square feet to square metres, using the conversion fact 1 inch = 2.54 cm (exactly).

Most people do not memorise the conversion ratios for area, but know 1 foot = 12 inches, and 1 m = 100 cm. We use these facts to get:

$$15\text{ ft}^2 = 15\ \cancel{\text{ft}^2} \times \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \times \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{1\text{ m}}{100\ \cancel{\text{cm}}} \times \frac{1\text{ m}}{100\ \cancel{\text{cm}}}$$

$$= 15 \times \frac{12^2 \times 2.54^2}{100^2}\text{ m}^2 = 1.3935456\text{ m}^2$$

Reflecting on this example we note the following:

- The unit 'square feet' can be written as ft^2, and we can think of this as ft $\times$ ft.
- Since 'ft' appears twice in the numerator, we need 'ft' twice in the denominator, and so the conversion fraction must be used twice.
- A different way of thinking about this would be to write all conversion factors as 'squared', as suggested by the second line of the calculation:

$$15\ \text{ft}^2 = 15\,\text{ft}^2 \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^2 \times \left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right)^2 \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2$$

  Since the exponent applies to everything inside the brackets, then we can cancel the units as before and continue with the second line of the original calculation.

Units and Conversion Facts – what you need to know

For the **metric system** you are expected to know:

- the unit names and their symbols
- the meaning of the prefixes and their symbols
- one of the following: 1 ml = 1cm^3, 1 litre = 1000cm^3

Since some **traditional units** are in common usage in Canada, you should be familiar with at least the following facts: ['=' means 'exactly equal to', and '$\approx$' means 'approximately equal to'.]

- 1 foot = 12 inches, 1 yard = 3 feet, 1 mile = 5280 feet
  - 1 inch = 2.54 cm (exactly, by definition), 1 yard $\approx$ 1 m, 1 mile $\approx$ 1.6 km
- 1 lb (pound) = 16 oz (ounces), 1 ton (US) = 2000 lbs
  - 2.2 lbs $\approx$ 1 kg, 1 ton $\approx$ 1 tonne
- 1 gallon = 4 quarts = 8 pints, 1 pint (US) = 16 fluid ounces, 1 pint (Imperial) = 20 fl oz
  - 1 pint $\approx$ ½ litre, 1 gallon (US) $\approx$ 3.8 litres, 1 gallon (Imperial) $\approx$ 4.5 litres

Practice Questions Set 3

[Answers to these questions can be found at the end.]

For the following questions, use the relationships given in this Review, and the method described in this section.

1) (a) 0.000047 kg = _____ mg (b) 178400 L = _____ m^3 (c) 4621500 mm = _____ km
(d) 4000 m^2 = _____ km^2 (e) 0.75 yd = _____ in (f) 0.0008 sq miles = _____ sq ft

2) James is thinking of buying a house and is looking for a house with a large yard. He works in Vancouver, and doesn't want a journey much longer than ½ hour. He visits several website of estate agents and sees the following properties.

(a) One house has a lot size of 12,936 sq ft and the brochure describes the lot as "almost 1/3 of an acre". Is this a reasonable claim? [Source: point2homes.com]

(b) A second house has a lot size of 1295 m^2. Is this bigger or smaller than the lot in (a)? [Source: homehuntersbc.com]

(c) A third house has a lot size of 362,593,440 sq ft and the brochure describes the lot as "an extra-large lot". Should he believe this? Suppose the lot were square, how long would the side-length of the square be? [Source: point2homes.com]

3) (a) Katherine needs 3 cubic metres of top soil for her new lawn. At the garden centre top soil is sold in cubic feet. How much should she buy?

(b) At the supermarket, prices are often shown in traditional units, but on the till receipt the price is shown in metric units. I buy some bananas marked 64 cents per lb. What price (in $/kg) should be shown on the till receipt?

(c) Speed limits in Canada are shown in km/h, but in the USA are given in mph. A Canadian visiting the US sees a speed limit of 55. Using the approximation 1 mile $\approx$ 1.6 km, convert this speed limit to km/h. To check the precision of this approximation, use the exact relationship 1 inch = 2.54 cm to convert the speed limit. Comment on your findings.

(d) Water is regarded as 'soft' if it has 1 to 4 grains per US gallon of calcium and magnesium ions, and 'hard' if it has 11 to 20 grains per US gallon. Although no research has confirmed health risks from hard water, recommendations for a max 110 mg/L for drinking water have been suggested. Using the conversion ratios 7000 grains = 1 pound, 1 oz = 28 g and 1 US gal = 3.8 L, convert 11 grains/US gal to mg/L, and comment on the value.

4) (a) Kitchen countertops are often made of granite, which has a density of 2.6 g/cm^3. Give this density in kg/m^3.

(b) The French V150 high speed train was named because the target speed was 150 m/s. The record achieved by the train in April 2007 was actually 159.7m/s. What speed is this in km/h and in mph.

(c) Natural crude oil seeps from the 1200 fissures in the ocean floor off the Californian coast. The rate of this seepage is about 0.37 mL/s from each fissure. What is the total number of litres per day of crude oil coming from these fissures? The famous Exxon Valdez oil spill in 1989 released about 40000 m^3 of oil according to official reports (although some argue it may have been as much as three times that amount). How many days of natural seepage from the Californian fissures would release that amount of oil into the ocean?

Answers

Practice Questions Set 1

1) (a) 1.6 m = 1.6×100 cm = 160 cm (b) 1.6 cm = $\frac{1.6}{100}$ m = 0.016 m

(c) 0.45 kg = 0.45×1000 g = 450 g (d) 0.25 L = 0.25×1000 mL = 250 mL

(e) 3.4 Mt = 3.4×1000000 t = 3,400,000 t

(f) 62.5 nm = $\frac{62.5}{1000\,000\,000}$ m = $\frac{62.5}{1000\,000\,000}\times1000$ mm = $\frac{62.5}{1000\,000}$ mm = 0.0000625 mm

(g) 1 h = 60 min = 60×60 s = 3600 s (h) 7 days = 7×24 h = $7\times24\times60$ min = 10080 min

(i) 500 min = 480 + 20 min = 8 h 20 min [$\frac{500}{60} = 8.33\overline{3}$; 8 full hours = 8×60 min = 480 min]

2) Width of rectangle = 60 cm = 0.6 m. Area of rectangle = $1.3\times0.6 = 0.78\text{ m}^2$
Length of rectangle = 1.3 m = 130 cm. Area of rectangle = $130\times60 = 7800\text{ cm}^2$
To show that these values are actually the same, we consider that 1 m = 100 cm, and so $1\text{ m}^2 = (100\times100)\text{ cm}^2 = 10{,}000\text{ cm}^2$. Therefore, $0.78\text{ m}^2 = 0.78\times10000\text{ cm}^2 = 7800\text{ cm}^2$.

3) (a) height of tall building: metres (m)
(b) mass of house key: grams (g)
(c) area of BC: square kilometres (km^2)
(d) capacity of mug: millilitres (ml or mL)
(e) volume of ball: cubic centimetres (cm^3)
(f) area of carpet: square metres (m^2)
(g) mass of passenger train carriage: tonnes (t)
(h) thickness of paper: micrometres (μm)
(i) area of door: square centimetres (cm^2)
(j) mass of full backpack: kilograms (kg)
(k) capacity of paddling pool: litres (l or L)
(l) volume of concrete driveway: cubic metres (m^3)

4) (a) 1 cm^3 = 1 ml, so 720 cm^3 = 720 ml = $\frac{720}{1000}$ l = 0.72 l.
(b) 1 m = 100 cm, so $1\text{ m}^3 = (100\times100\times100)\text{cm}^3 = 1{,}000{,}000\text{ cm}^3$ [Imagine finding the volume of a cube with side length 1 m, which is a side length of 100 cm.]
So, $0.0032\text{ m}^3 = 0.0032\times1{,}000{,}000\text{ cm}^3 = 3{,}200\text{ cm}^3$.
(c) Using the relationship in (b), $1\text{cm}^3 = \frac{1}{1000000}\text{m}^3$, so $350000\text{ cm}^3 = \frac{350000}{1000000}\text{m}^3 = 0.35\text{ m}^3$.
Using the relationship in (a), 1 cm^3 = 1 ml, so 350000 cm^3 = 350000 ml = $\frac{350000}{1000}$ l = 350 l.

5) '4 metres squared', $(4\text{ m})^2$, describes a square of side-length 4 metres (which has an area of $4\text{ m}\times4\text{ m} = 16\text{ m}^2$), whereas '4 square metres', 4 m^2, describes an amount of area (but not its shape).

6) (a) I completed the 10 km fun run in exactly 2 hours. I averaged 5 km/h.
(b) After working 4 hours, Tim is paid \$88. He is paid 22 \$/h.
(c) A summer worker planted 6000 trees and was paid \$1800. She was paid 0.3 \$/tree.
(d) A hockey player was on the ice for 1080 minutes this season, in which he played 45 games. His stats listed "time on ice" as 24 min per game.

7) It is true that the USA's public debt is almost ten times that of Canada, since 10 × \$1,200 billion = \$12,000 billion, which is just a little more than \$11,600 billion. These values are the total public debt in each country.

It is also true that \$34,900 and \$36,600 are similar amounts. However, these are the debt per capita rather than the total debt; it means that each adult and each child in the country would need to give about \$35,000 to the government in order to pay off all the debt! It would have been much clearer if this had been indicated in the units by writing \$34,900/capita and \$36,600/capita.

We can confirm this interpretation of the numbers as follows:

The population of Canada is about 34.5 million, giving a debt of $\frac{1200\text{ billion}}{34.5\text{ million}} = \frac{1200\,000\text{ million}}{34.5\text{ million}} \approx \$34\,800\text{/capita}$. The population of the USA is about 315 million, giving a debt of $\frac{\$11600\text{ billion}}{315\text{ million}} = \frac{\$11600\,000\text{ million}}{315\text{ million}} \approx \$36\,800\text{/capita}$.
The slight differences between the calculated values and those quoted could be due to slightly different values for the populations, since these are constantly changing.

Practice Questions Set 2

1) (a) 8 ft = 8×12 in = 96 in (b) 26400 ft = $\frac{26400}{5280}$ miles = 5 miles (c) 4 gal = 4×8 pt = 32 pt
(d) 32 oz = $\frac{32}{16}$ lb = 2 lb (e) 5 ft 7 in = $5\times12+7$ in = 67 in (f) 52 oz = 48+4 oz = 3 lb 4 oz

2) (a) 6 sq yd = $6\times3\times3$ sq ft = 54 sq ft (b) 720 sq in = $\frac{720}{12\times12}$ sq ft = 5 sq ft
(c) 135 cu ft = $\frac{135}{3\times3\times3}$ cu yd = 5 cu yd

3) (a) 12 fl oz $\approx 12\times30$ ml = 360 ml (b) 60 kg $\approx 60\times2.2$ lb = 132 lb
(c) 55 miles $\approx 55\times1.6$ km = 88 km (d) 420 g $\approx \frac{420}{28}$ oz = 15 oz
(e) 6 ft = 2 yd $\approx$ 2 m or 6 ft $\approx 6\times30$ cm = 180 cm = 1.8 m
(f) 500 L $\approx \frac{500}{4.5}$ gal $\approx$ 110 gal (Imperial) or 500 L $\approx \frac{500}{3.8}$ gal $\approx$ 130 gal (US)

Practice Questions Set 3

1) (a) $0.000047\text{ kg} = 0.000047\ \cancel{\text{kg}} \times \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \times \frac{1000\text{ mg}}{1\ \cancel{\text{g}}} = 47\text{ mg}$

(b) $178400\text{ L} = 178400\ \cancel{\text{L}} \times \frac{1000\ \cancel{\text{cm}^3}}{1\ \cancel{\text{L}}} \times \left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right) \times \left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right) \times \left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right) = 178.4\text{ m}^3$

(c) $4621500\text{ mm} = 4621500\ \cancel{\text{mm}} \times \frac{1\ \cancel{\text{m}}}{1000\ \cancel{\text{mm}}} \times \frac{1\text{ km}}{1000\ \cancel{\text{m}}} = 4.6215\text{ km}$

(d) $4000\text{ m}^2 = 4000\ \cancel{\text{m}^2} \times \left(\frac{1\text{ km}}{1000\ \cancel{\text{m}}}\right) \times \left(\frac{1\text{ km}}{1000\ \cancel{\text{m}}}\right) = 0.004\text{ km}^2$

(e) $0.75\text{ yd} = 0.75\ \cancel{\text{yd}} \times \frac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}} \times \frac{12\text{ in}}{1\ \cancel{\text{ft}}} = 27\text{ in}$

(f) $0.0008\text{ sq miles} = 0.0008\ \cancel{\text{sq miles}} \times \left(\frac{5280\text{ ft}}{1\ \cancel{\text{mile}}}\right) \times \left(\frac{5280\text{ ft}}{1\ \cancel{\text{mile}}}\right) = 22\,302.72\text{ sq ft}$

2) (a) $12936 \text{ sq ft} = 12936 \ \cancel{\text{sq ft}} \times \dfrac{1 \ \cancel{\text{mile}}}{5280 \ \cancel{\text{ft}}} \times \dfrac{1 \ \cancel{\text{mile}}}{5280 \ \cancel{\text{ft}}} \times \dfrac{640 \text{ acre}}{1 \ \cancel{\text{sq mile}}} \approx 0.297 \text{ acre}$ and $\frac{1}{3} \approx 0.333$, so the claim is reasonable (although a bit generous!).

(b) $1295 \text{ m}^2 \approx 1295 \ \cancel{\text{m}^2} \times \dfrac{100 \ \cancel{\text{cm}}}{1 \ \cancel{\text{m}}} \times \dfrac{100 \ \cancel{\text{cm}}}{1 \ \cancel{\text{m}}} \times \dfrac{1 \text{ ft}}{30 \ \cancel{\text{cm}}} \times \dfrac{1 \text{ ft}}{30 \ \cancel{\text{cm}}} \approx 14000 \text{ sq ft}$, so this is bigger than the lot in (a). [Note: since 30 cm is only an approximation of 1 foot, we cannot give the number of square feet to more than 2 significant digits. For more about precision see the "Approximation and Precision" review sheet.]

(c) $362593440 \text{ sq ft} = 362593440 \ \cancel{\text{sq ft}} \times \dfrac{1 \text{ mile}}{5280 \ \cancel{\text{ft}}} \times \dfrac{1 \text{ mile}}{5280 \ \cancel{\text{ft}}} \approx 13 \text{ sq miles}$

$13 \text{ sq miles} \approx 13 \ \cancel{\text{sq miles}} \times \dfrac{1.6 \text{ km}}{1 \ \cancel{\text{mile}}} \times \dfrac{1.6 \text{ km}}{1 \ \cancel{\text{mile}}} \approx 33 \text{ km}^2$ If this were true, then the lot would have the same area as a square of side-length 3.6 miles or 5.7 km. Since James is looking at houses quite close to Vancouver, we should not believe this information!

3) (a) $3 \text{ m}^3 \approx 3 \ \cancel{\text{m}^3} \times \dfrac{100 \ \cancel{\text{cm}}}{1 \ \cancel{\text{m}}} \times \dfrac{100 \ \cancel{\text{cm}}}{1 \ \cancel{\text{m}}} \times \dfrac{100 \ \cancel{\text{cm}}}{1 \ \cancel{\text{m}}} \times \dfrac{1 \text{ ft}}{30 \ \cancel{\text{cm}}} \times \dfrac{1 \text{ ft}}{30 \ \cancel{\text{cm}}} \times \dfrac{1 \text{ ft}}{30 \ \cancel{\text{cm}}} \approx 110 \text{ cu ft}$

Katherine should buy 110 cubic feet of top soil.

(b) $64 \text{ cents/lb} \approx \dfrac{64 \ \cancel{\text{cents}}}{1 \ \cancel{\text{lb}}} \times \dfrac{2.2 \ \cancel{\text{lb}}}{1 \text{ kg}} \times \dfrac{1 \ \$}{100 \ \cancel{\text{cents}}} = 1.408 \ \$/\text{kg}$ The till receipt will probably show \$1.41/kg.

(c) $55 \text{ mph} \approx \dfrac{55 \ \cancel{\text{miles}}}{1 \text{ h}} \times \dfrac{1.6 \text{ km}}{1 \ \cancel{\text{mile}}} = 88 \text{ km/h}$

$$55 \text{ mph} = \frac{55 \ \cancel{\text{miles}}}{1 \text{ h}} \times \frac{5280 \ \cancel{\text{ft}}}{1 \ \cancel{\text{mile}}} \times \frac{12 \ \cancel{\text{in}}}{1 \ \cancel{\text{ft}}} \times \frac{2.54 \ \cancel{\text{cm}}}{1 \ \cancel{\text{in}}} \times \frac{1 \ \cancel{\text{m}}}{100 \ \cancel{\text{cm}}} \times \frac{1 \text{ km}}{1000 \ \cancel{\text{m}}}$$

$$= \frac{(55 \times 5280 \times 12 \times 2.54) \text{ km}}{(100 \times 1000) \text{ h}} = 88.51392 \text{ km/h}$$

The approximation is within 0.6% of the exact value ($\frac{88}{88.51392} \times 100 = 99.42\%$), which shows that it is very good for most practical purposes.

(d) $$20 \text{ grains per US gallon} = \frac{11 \ \cancel{\text{grains}}}{1 \ \cancel{\text{gal}}} \times \frac{1 \ \cancel{\text{gal}}}{3.8 \text{ L}} \times \frac{1 \ \cancel{\text{lb}}}{7000 \ \cancel{\text{grain}}} \times \frac{16 \ \cancel{\text{oz}}}{1 \ \cancel{\text{lb}}} \times \frac{28 \ \cancel{\text{g}}}{1 \ \cancel{\text{oz}}} \times \frac{1000 \text{ mg}}{1 \ \cancel{\text{g}}}$$

$$= \frac{(11 \times 16 \times 28 \times 1000) \text{ mg}}{(3.8 \times 7000) \text{ L}} \approx 185 \text{ mg/L}$$

This suggests that all areas with hard water have a concentration above the recommended maximum!

4) (a) $2.6\ \text{g/cm}^3 = \dfrac{2.6\ \cancel{\text{g}}}{1\ \cancel{\text{cm}^3}} \times \dfrac{1\ \text{kg}}{1000\ \cancel{\text{g}}} \times \dfrac{100\ \cancel{\text{cm}}}{1\ \text{m}} \times \dfrac{100\ \cancel{\text{cm}}}{1\ \text{m}} \times \dfrac{100\ \cancel{\text{cm}}}{1\ \text{m}} = 2600\ \text{kg/m}^3$

(b) $159.7\ \text{m/s} = \dfrac{159.7\ \cancel{\text{m}}}{1\ \cancel{\text{s}}} \times \dfrac{1\ \text{km}}{1000\ \cancel{\text{m}}} \times \dfrac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \dfrac{60\ \cancel{\text{min}}}{1\ \text{h}} = 574.92\ \text{km/h} \approx 575\ \text{km/h}$

$575\ \text{km/h} \approx \dfrac{574.92\ \cancel{\text{km}}}{1\ \text{h}} \times \dfrac{1\ \text{miles}}{1.6\ \cancel{\text{km}}} \approx 360\ \text{mph}$

(c) $\text{Total seepage} = 1200 \times 0.37\ \text{mL/s} = \dfrac{444\ \cancel{\text{mL}}}{1\ \cancel{\text{s}}} \times \dfrac{1\ \text{L}}{1000\ \cancel{\text{mL}}} \times \dfrac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \dfrac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \dfrac{24\ \cancel{\text{h}}}{1\ \text{day}}$

$= \dfrac{444 \times 60 \times 60 \times 24\ \text{L}}{1000\ \text{day}} = 38361.6\ \text{L/day} \approx 38\,000\ \text{L/day}$

$40\,000\ \text{m}^3 = 40\,000\ \cancel{\text{m}^3} \times \dfrac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}} \times \dfrac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}} \times \dfrac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}} \times \dfrac{1\ \text{L}}{1000\ \cancel{\text{cm}^3}} = 40\,000\,000\ \text{L}$

$\dfrac{40\,000\,000}{38\,000} \approx 1050$, so it would take about 1050 days (close to 3 years) of natural seepage to release as much oil as the man-made disaster.