Phonological Events

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1 Introduction

One of the major innovations within post-SPE generative phonology has been the development of frameworks where phonological units are organised in a non-linear fashion. Taking autosegmental phonology (Goldsmith 1976) as our main exemplar of such frameworks, we wish to address the following question: What is the appropriate interpretation of autosegmental representations? There is, of course, a further question about what we mean by *interpretation*: formal, phonetic or computational interpretation? Although we will concentrate on the first of these, we believe that all three aspects should be regarded as closely interconnected and mutually constraining.

The question of interpreting autosegmental representation has in fact been recently posed by Sagey (1988), and we shall take her proposal as our starting point. While it is uncontroversial to suppose that the relationship between units on a given autosegmental tier is one of temporal precedence, Sagey claims that it is more problematic to pin down what is meant by association between tiers. She argues, cogently we believe, that if association is taken to be a relationship of simultaneity between durationless units, then standard analyses of complex segments and gemination lead to logical inconsistency. Instead, association should be taken as temporal *overlap* between units with duration.

We begin with a review of Sagey's proposals, observing that she adopts an ontology based on points, where intervals are defined as sets of points. We argue that this leads to a number of formal, phonetic, philosophical and cognitive problems, and propose an alternative approach using an ontology based on intervals. In section 2 we define an *event* to be a compound entity consisting of an interval together with a property, and provide axioms governing the overlap and precedence relations which hold between pairs of such events. The resulting ontology, we argue, provides a natural framework within which to model important relationships between phonological gestures. Section 3 begins with a presentation of *event structures*, which are collections of events and constraints. We show, with a variety of illustrations, how event structures can be used to formalize the various components of multi-tiered, hierarchical autosegmental representations. We also discuss their close relationship to the notion of a gestural score. It should be stressed at the outset that this article concerns autosegmental representations, and not the rules which are presumed to manipulate them. Due to the expository goals of this paper we have not attempted to carry out a detailed analysis of a large body of phonological data, however we acknowledge that this is an important task and it is one that we intend to undertake in future work.

Deriving the No-Crossing Constraint

Sagey defines three relations on temporal units: simultaneity, precedence and overlap. Certain facts about the first two relations (and presumably the third also) are taken to be 'included in our knowledge of the world' (p.110). We begin with a brief review of these facts.

Temporal overlap is a two-place relation which is reflexive, symmetric and nontransitive. If we employ the notation $x \circ y$ for the statement 'x overlaps y' then these facts about overlap can be stated as follows:

(1) a. For any x, $x \circ x$ *overlap is reflexive*
b. For any x and y, if $x \circ y$ then $y \circ x$ *overlap is symmetric*

If overlap were transitive, a third statement would be necessary:

(2) For any x, y and z, if $x \circ y$ and $y \circ z$ then $x \circ z$

However, if this were the case we would be back where we began, where association was conceived as simultaneity. Since overlap is nontransitive, we simply omit this statement. (Note that this does not preclude the relation expressed in (2) from holding for a particular choice of x, y and z; it is just not guaranteed to hold for all such choices.)

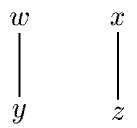
Above we described the relation holding between members of a tier as 'temporal precedence'. By this we meant strict linear precedence, which is an irreflexive, asymmetric and transitive relation.[1] We adopt the notation $x \prec y$ to express the statement 'x precedes y', and write the following expressions (where negation ($\neg$) is taken to have wider scope than $\prec$ and $\circ$):

(3) a. For any x, $\neg x \prec x$ *precedence is irreflexive*
b. For any x and y, if $x \prec y$ then $\neg y \prec x$ *precedence is asymmetric*
c. For any x, y and z, if $x \prec y$ and $y \prec z$ then $x \prec z$ *precedence is transitive*

Perhaps surprisingly, the properties expressed above about overlap and precedence are inadequate in a crucial way. Consider the statement: '$x \prec y$ and $x \circ y$'. Clearly, we want this to be inconsistent, given the intended interpretations of $\prec$ and $\circ$. However, we cannot demonstrate this from what we have said so far. Thus, to express the mutual exclusiveness of overlap and precedence, a further statement is necessary:

(4) For any x and y, if $x \prec y$ then $\neg x \circ y$

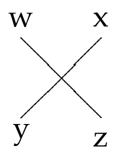
At this point, we seem to have enough machinery to interpret an autosegmental diagram such as (5).

(5)


A line which connects two points, say those labeled w and y, is interpreted as claiming that there is an overlap relation holding between events w and y, while horizontal alignment of two points on the page, say w appearing to the left of x, is interpreted as claiming that a relation of precedence holds between w and x. That is, (5) depicts a situation which we can describe in our notation as follows:

(6) $w \prec x$, $y \prec z$, $w \circ y$ and $x \circ z$.

Now let us consider the situation shown in (7), where two association lines cross:

(7)


We interpret this as shown in (8):

(8) For some w, x, y and z, (i) $w \prec x$ and $y \prec z$ and (ii) $w \circ z$ and $x \circ y$

However, none of the above facts about overlap and precedence rule out (8) as ill-formed, and a further statement about the relationship between overlap and precedence is therefore necessary. This is given in (9).

(9) For any w, x, y, z, if $w \prec x$ and $y \prec z$ and $x \circ y$ then $w \prec z$.

In order to help visualise the constraint that is imposed in (9), we adopt Sagey's graphical conventions for representing intervals as labeled time-line segments:

(10) [PICTURE NOT AVAILABLE]

Given the statements in (4) and (9) about the relationship between overlap and precedence, the no-crossing constraint can be derived. Suppose we have $w \prec x$ and $y \prec z$. From (9) we know that $w \prec z$ and then from (4) that $\neg w \circ z$. The key point here is that the no-crossing constraint does not follow from the definitions of overlap and precedence alone, but from additional statements about their interrelationship.

One apparent virtue of Sagey's approach is that she does not need to stipulate these additional properties. In fact, *all* of the properties of overlap, precedence and their inter-relationship stated above follow from Sagey's conception of intervals as a collection of points. The definitions in (11) are revised versions of those given by Sagey, who employs the notation 'All P(x)' to refer to the collection of points in an interval x and 'Some P(x)' to refer to a particular point x. Given her view of an interval as a set of points, there is clearly no distinction to be drawn between x and 'All P(x)'. Moreover, the precise meaning of 'P(x)' is unclear. Instead we use standard set-theoretical notation to talk about the elements of a set, and formulate the following definitions:

(11) a. For two intervals x and y, we write $x \prec y$ iff for all $p \in x$ and for all $q \in y$, $p < q$.

b. For two intervals x and y, we write $x \circ y$ iff for some $p \in x$ and for some $q \in y$, $p = q$.

Given that $<$ is a strict linear ordering on points, it is a relatively straightforward matter to show that (1), (3), (4) and (9) above follow from these definitions and therefore do not require independent statement. This would appear to be a desirable state of affairs, given the economy of statement and simplicity of (11). However, defining intervals in terms of points is questionable from both a philosophical and a cognitive viewpoint.

Although it is a deeply rooted part of our current scientific outlook to regard time as being composed of instants and collections of instants, it has nevertheless been argued by philosophers such as Russell that viewing time as consisting of extended periods which admit ever-finer subdivisions is closer to our pretheoretic intuitions. From a cognitive standpoint, the definitions in (11) are also rather implausible. They suggest that in order for an agent to verify a statement of precedence between two intervals containing an infinity of points, she would have spend forever comparing the points in a pairwise manner; a similarly non-terminating procedure would be required to falsify a statement of overlap between two intervals. Even if one argued that such intervals contained only a finite number of points, the cognitive processing required would be dependent upon the size of the intervals. This is contrary to the seemingly uncontroversial claim that it should take constant time to judge the precedence or overlap of arbitrarily sized intervals.

The above definitions could be rescued from this criticism by referring to the *endpoints* of intervals. Let us assume, as before, that $<$ is a strict linear ordering on points, and also that 'max(x)' denotes the maximal (i.e. last) element of interval x with respect to $<$, and that 'min(x)' denotes the minimal (i.e. first) element of x.

(12) a. $x \prec y$ iff $max\,(x) < min\,(y)$

b. $x \circ y$ iff $max\,(x) > min\,(y)$ and $max\,(y) > min\,(x)$

For the interval endpoints to be specifiable in a way that is independent of the size of the interval (*i.e.* the number of points it contains, whether finite or infinite), they must be basic to the definition of the interval. In other words, the interval must be defined in terms of its endpoints, rather than as the set of points it contains—for example, as $\{t \mid 3.42 \leq t \leq 3.96\}$, where the numbers represent seconds since the beginning of the utterance. Given the endpoints, it is then a simple matter to determine whether a point is contained in the interval.

However, this position runs into a number of difficulties. First, it is usually difficult to assign a determinate boundary (either perceptually or instrumentally) to the phonetic instantiation of a phonological event. We can be certain about the 'central area' of, say, an interval of nasality or friction in an utterance, but as we near either extremity of such an interval it becomes less certain whether or not a particular point is included in the interval. Consequently, it would seem desirable to allow for a degree of indeterminacy in the location of interval endpoints.

Moreover, even if it were possible from a phonetic point of view to demarcate precisely the beginning and endpoints of some particular event such as voicing, it is highly implausible that one would want to treat such boundaries as part of the phonological specification of a feature or autosegment. This is partially acknowledged by Sagey's claim that "the points of time within a feature or x-slot are accessible only at the late level of phonetic implementation, . . ., they are not manipulable or accessible by phonological rules" (1986:294). Yet (as also pointed out by Hammond 1988:323) this is difficult to reconcile with the fact that points are fundamental to Sagey's ontology.

A related issue is that the phonetic properties of a given point can only be specified in terms of an interval (possibly very small) which contains that point. Thus on Sagey's approach, one first has to construct intervals from points, and only then attach certain properties to these intervals, a two-stage process. This situation is necessitated by a further fact. If a feature is simply considered to be an interval and nothing more, then we could not adequately accommodate a situation where two *distinct* features occupy one and the same interval, because they would then be indistinguishable.

These problems do not arise if intervals are taken as basic to phonology. We believe that Sagey's proposals represent a big step in this direction, but that they do not go far enough. It is perhaps interesting to note that the ontological shift from points to intervals is not new to linguistics; for example, a similar move was made in linguistic semantics by Bennett & Partee (1972).

Recall that intervals have properties attached to them. From now on an interval and a property will be regarded as two aspects of a single unit. When they are bundled together in this way, the result is usually referred to as an *event* (cf. van Benthem 1983:113).

2 Events

The properties of overlap and precedence stated in the last section are gathered together in (13) below. However, from now on, we use the variables w, x, y and z to refer to events rather than intervals. Note that it is unnecessary, and somewhat misleading, to portray events as labeled time-line segments. Events are basic entities in our ontology, having no internal structure other than a particular stated property, and thus can be represented quite adequately by points in our diagrams. As we saw in the preceding section, we can adopt interpretive conventions for standard autosegmental notation whereby association lines correspond to temporal overlap, and left-to-right arrangement on the page corresponds to temporal precedence. Consequently, phonologists can use the usual graphical notation for autosegments and association, while still maintaining the view (if they wish) that autosegments have internal duration.

We said above that an event has a property. This property will correspond to a feature or a gesture. The notion of gesture that we have in mind corresponds broadly to that found in Browman & Goldstein (1986, 1989), Ewen (1986), Lass (1983), Pierrehumbert & Beckman (1988). The latter state '[the elements] could be tones or phonemes, but also demisyllables, articulatory commands, or whatever' (153). Further questions can obviously be raised as to whether such properties play a contrastive role in a phonological system, or whether they are the phonetic realizations of phonological properties. Despite the fact that this is a central issue in developing a detailed theory of event-based phonology, it is one that we cannot address adequately within the confines of this paper, and will therefore sidestep. Our terminology 'phonological events' is intended to be neutral with respect to the phonology/phonetics distinction.

2.1 Axioms for Events

Summarizing from §1, we have the following collection of statements governing a set E of phonological events:[2]

(13) a. For any event $x \in E$, $x \circ x$.
Overlap is reflexive (every event overlaps itself).

b. For any events $x, y \in E$, if $x \circ y$ then $y \circ x$.
Overlap is symmetric (overlapping an event implies being overlapped by it).

c. For any events $x, y \in E$, if $x \prec y$ then $\neg y \prec x$.
Precedence is asymmetric (preceding an event implies not being preceded by it).

d. For any events $x, y \in E$, if $x \prec y$ then $\neg x \circ y$.
Precedence is disjoint from overlap (preceding an event implies not overlapping it).

e. For any events w, x, y and $z \in E$, if $w \prec x$, $x \circ y$, and $y \prec z$ then $w \prec z$.
If one event precedes a member of an overlapping pair of events, and a second event follows the other member of that pair, then the first event precedes the second.

This collection of statements is *minimal*, in the sense that none can be inferred from any combination of the others, and constitute the basic assumptions made. Following standard mathematical practice we will call them *axioms*. Three consequences of these axioms, already discussed in §1, are listed in the Appendix.

We will also presume the following rule of inference:

(14) **Modus Ponens:** Given a proposition A, and the conditional expression 'if A then B', infer B.

The reasons for stating axioms are numerous. For example, given a collection of events and certain information about the overlap and precedence relations existing between various units, it is possible to deduce further information. Thus, if we know that a segment **p** precedes a segment **i** and that **i** precedes **n**, we can infer that **p** precedes **n** using (13e), equating x and y. Put slightly differently, the composite statement '**p** $\prec$ **i**, **i** $\prec$ **n** and **p** $\prec$ **n**' contains redundant information, given the transitivity of the relation $\prec$, and so we can abbreviate it by omitting '**p** $\prec$ **n**'.

In addition, it is possible to tell if a set of overlap and precedence statements is consistent; we use the axioms and the inference rule to derive all that can be derived and check that no contradictory statements are present. To summarise then, writing expressions of this kind admits *inference*, *abbreviation* and *consistency checking*. The axiomatic approach is not new to phonology, and has been explored by such linguists as Bloomfield (1926), Bloch (1948), Greenberg (1959) and Batóg (1967). However, there have been few attempts to axiomatize autosegmental phonology.

2.2 Defining Inclusion

Now that axioms for overlap have been provided, it is possible to define temporal inclusion. In fact, inclusion and overlap are interdefinable (van Benthem 1985:35–6), as shown in the following definitions. (The statement '$x \sqsubseteq y$' should be read: x is included in y.)

(15) a. For all $x, y \in E$, $x \sqsubseteq y$ iff every $z \in E$ which overlaps x also overlaps y.
b. For all $x, y \in E$, $x \circ y$ iff there is a $z \in E$ which is included in both x and y.

From this it follows that inclusion is a reflexive and transitive relation. What about symmetry? In general, it will be the case that if $x \sqsubseteq y$ then $\neg x \sqsupseteq y$ (i.e. y is not included in x). Nevertheless, we want to allow the possibility that both $x \sqsubseteq y$ and $x \sqsupseteq y$ hold, corresponding to our intuitive notion of simultaneity. The abbreviatory notation we adopt here is '$x \leftrightarrow y$', to be read: x and y are simultaneous (or coterminous). Of course, two events can be simultaneous without being identical, so we favour the use of $\leftrightarrow$ over $=$, which Sagey rightly adopts for points and intervals. Here, then, is the definition of simultaneity:

(16) For all $x, y \in E$, $x \leftrightarrow y$ iff $x \sqsubseteq y$ and $x \sqsupseteq y$

A direct consequence is that simultaneity is an equivalence relation, that is, reflexive, symmetric and transitive.

Although we will be mainly concerned with exploring the interpretation of association as overlap, we expect that the interpretation of association as inclusion or as simultaneity will be useful on occasion, particularly in those cases where the transitivity property is required (e.g. when two autosegments on distinct tiers, linked to the same x-slot, are interpreted as co-articulated, or where association is used to encode hierarchy and 'feature percolation', Clements 1985:250). The inclusion relation may also be useful to express constraints on the spreading of autosegments: if $x \sqsubseteq y$ then x cannot 'spread' beyond the limits of y.

2.3 Homogeneity & Convexity

Now that we have introduced the notion of inclusion, we can ask about the subevents which might be included within a given event. Take, for example, a **[+nasal]** event ϵ. It is plausible to suppose that all the phonologically relevant subevents of ϵ also have the property of being nasal; that is, the property of nasality is uniformly spread over the whole of ϵ. In this case, we say that the event is *homogeneous*.

By contrast, we might want to claim that a **[+stop]** event ϵ can be further analyzed as a **[+closure]** event ϵ_1 followed by a **[+release]** event ϵ_2. We can now do so, with a statement to the effect that ϵ includes both ϵ_1 and ϵ_2. Events which contain distinct subparts in this way will be termed *heterogeneous*.[3]

A related issue arises when we consider a phenomenon such as vowel harmony. We would like to be able to say that the distinctive features common to all of the harmonizing vowels come from a single source, namely the properties of a single event ϵ which overlaps each vowel slot. However, let us consider a sequence **V C V**, where the two **V**s harmonize, say, for the feature **[+back]**. This means, in particular, that a **[+back]** event overlaps both of the **V** events. Does it also overlap **C**? At a phonological level, we do not want to be committed to such a consequence (although it is one which follows on Sagey's account[4]), since the feature **[+back]** might be either inappropriate or false for the **C**.

More generally, we are concerned here with a characteristic of events which has been termed 'convexity'. An event ϵ is *convex*, by definition, if it satisfies the following condition:

(17) For all x_1, x_2, x_3, if $x_1 \prec x_2$, $x_2 \prec x_3$, $x_1 \circ \epsilon$, and $x_3 \circ \epsilon$ then $x_2 \circ \epsilon$.

That is, if ϵ overlaps two events x_1 and x_3, then it also overlaps any other event x_2 which intervenes between x_1 and x_3.

An event not satisfying this condition it will be called *non-convex*. We will admit into our framework events of both sorts. Thus, the harmonizing **V**s in our immediately preceding example will be part of a non-convex **[+back]** event. This dichotomy allows the local/long-distance spreading distinction (e.g. Hoberman 1988) to be represented.

2.4 Immediate Precedence

In the previous section we saw that overlap and inclusion are interdefinable, and it is essentially an issue of convenience which we take to be basic. A similar situation holds for precedence and a new relation called *immediate precedence*, written $\prec^\circ$. Like precedence, immediate precedence is irreflexive and asymmetric, but unlike precedence it is *intransitive*. In other words, if $x \prec^\circ y$ and $y \prec^\circ z$, then it cannot be the case that $x \prec^\circ z$. Immediate precedence can be defined in terms of $\prec$ as follows:

(18) For all $x, y \in E$, $x \prec^\circ y$ iff $x \prec y$ and there is no $z \in E$ such at $x \prec z \prec y$.

In what follows, we will sometimes find it useful to present relations as sets of ordered pairs. For example, if x and y are the only elements of some set E_1 such that $x \prec_1^\circ y$, then we can exhaustively characterize $\prec_1^\circ$ as the set $\{\langle x, y\rangle\}$. Suppose moreover that $\prec_2^\circ$ is a relation over the set E_2 such that $\prec_2^\circ = \{\langle y, z\rangle\}$. We can now build an relation $\prec^\circ$ over the set $E = E_1 \cup E_2$ by forming the union of the previous relations; that is, we have $\prec^\circ = \prec_1^\circ \cup \prec_2^\circ = \{\langle x, y\rangle, \langle y, z\rangle\}$.

Notice, also, that we can construct the ordering $\prec$ as the so-called transitive closure of $\prec^\circ$: in the case at hand it is the set $\{\langle x, y\rangle, \langle y, z\rangle, \langle x, z\rangle\}$. More generally, whenever we have a relation $\prec^\circ$ of immediate precedence, we can construct its counterpart $\prec$ by adding the requisite ordered pairs in the manner indicated below:

(19) If $\prec^\circ$ is a relation over E, then its *transitive closure* $\prec$ is the smallest set such that
(i) if $\langle x, y\rangle$ is in $\prec^\circ$ then $\langle x, y\rangle$ is in $\prec$, and
(ii) if $\langle x, y\rangle$ is in $\prec$ and $\langle y, z\rangle$ is in $\prec$, then $\langle x, z\rangle$ is in $\prec$.

It is not hard to see that the transitivity of precedence makes it awkward to talk about the *adjacency* of two events, for the truth of $x \prec y$ may well hide a multitude of events which intervene between x and y. This relation is fundamental to the definition of melodies in §3.1.

There are two more definitions relating to precedence which will be useful in §3, the first concerning linearity, and the second concerning boundedness.

(20) A precedence relation $\prec$ over a set E is a *linear ordering* if for all $x, y \in E$, $x \prec y$ or $x \succ y$ or $x = y$.

(21) An element x of E is $\prec$*-maximal* iff for all $y \in E$, either $y \prec x$ or $y = x$, and analogously for $\prec$*-minimal*.

3 Event Structures

The preceding discussion has led us towards what we have termed phonological events, corresponding indirectly in some way to small pieces of speech. An event is an interval together with a property, and this compound entity will be formalized as an ordered pair. The first element is an interval ι, and the second is a property π which obtains for that interval; such a pair will be written '$\iota : \pi$'.

The next step is to show how the phonological structure of melodies, words and phrases might be expressed using events. A synopsis of the strategy is simple; we allow π to represent a collection of events and constraints. In other words, our event-based analysis will be recursive, in the sense that properties of events can either be basic, or (more commonly) can be expressed as further event structures.

3.1 Melodies and Autosegmental Tiers

We start by defining the notion of a melody.

(22) A *melody* is an ordered pair $\tau = \langle E, \prec^\circ \rangle$, where
(i) E is a set of events,
(ii) $\prec^\circ$ is an irreflexive and asymmetric relation over E, and
(iii) E contains unique maximum and minimum elements with respect to $\prec$, denoted $max(\tau)$ and $min(\tau)$ respectively.

If $\tau = \langle E, \prec^\circ \rangle$, we shall sometimes refer to E as the *event set* of τ.

Suppose now that we want to formally characterize a sequence of three tones **L H L** on a tone tier as a melody $\tau = \langle E, \prec^\circ \rangle$. E will consist of a set of events $\iota{:}T$, where ι: is an interval, and T is a tonal property. More specifically, let us put $E = \{ \iota_1{:}\mathbf{L}, \iota_2{:}\mathbf{H}, \iota_3{:}\mathbf{L} \}$. The second component of τ is a set of pairs which expresses the immediate precedence relations holding between the tone events. Assuming the sequence indicated above, we put $\prec^\circ = \{ \langle \iota_1, \iota_2 \rangle, \langle \iota_2, \iota_3 \rangle \}$.

As a notational convenience, we will sometimes allow ourselves to encode the immediate precedence information as a list-like presentation of the set E. For example, our tone melody $\langle \{ \iota_1{:}\mathbf{L}, \iota_2{:}\mathbf{H}, \iota_3{:}\mathbf{L} \}, \{ \langle \iota_1, \iota_2 \rangle, \langle \iota_2, \iota_3 \rangle \} \rangle$ will be abbreviated to $[\iota_1{:}\mathbf{L}, \iota_2{:}\mathbf{H}, \iota_3{:}\mathbf{L}]$ or, even more tersely, as $[\mathbf{L}, \mathbf{H}, \mathbf{L}]$.

So far, nothing prevents the following from being an admissible tone melody: $[\iota_1{:}\mathbf{H}, \iota_2{:}\mathbf{ka}, \iota_3{:}\mathbf{+wide}]$. Intuitively, given a particular kind of tier (such as a tone tier), there is a strong restriction on the kind of autosegments which it is allowed to contain. This restriction can be encoded in the formalism in a natural way by employing *types*: events, melodies and (as will be seen later) event structures will all be typed. In our notation, types are made explicit as subscripts, as the following example illustrates. (Here, **t** is the type of tone tiers, **u** is the type of tone-bearing unit tiers, and VEL is the type of velic gestures.)

(23) a. $[\iota_1{:}\mathbf{L_t}, \iota_2{:}\mathbf{H_t}, \iota_3{:}\mathbf{L_t}]_\mathbf{t}$
b. $[\iota_1{:}\mathbf{H_t}, \iota_2{:}\mathbf{ka_u}, \iota_3{:}\mathbf{+wide_{VEL}}]_\mathbf{t}$

Let us therefore impose the condition that melodies should be *consistent*, in the sense that the type of a melody must agree with the types of all the events it contains. While the melody in (23a) meets this condition, the melody $\tau_\mathbf{t}$ in (23b) is inconsistent because it has the type of a tone tier, yet contains events of incompatible types, namely **u** and VEL .

Having introduced types, we should point out that our use of the term *tier* above is to be interpreted as synonymous with the notion of type; in particular, the statements 'melody τ is on tier θ' and 'melody τ has type θ' are intended to be equivalent.

The final step in developing the formalism for melodies is to specify how larger melodies can be related to smaller ones. The concatenation operation is used:

(24) The *concatenation* of two melodies τ_1 and τ_2 of the same type θ, where $\tau_1 = \langle E_1, \prec_1^\circ \rangle_\theta$ and $\tau_2 = \langle E_1, \prec_2^\circ \rangle_\theta$, is a third melody of type θ, namely $\tau = \langle E_1 \cup E_2, \prec_1^\circ \cup \prec_2^\circ \cup \{ \langle max(\tau_1), min(\tau_2) \rangle \} \rangle_\theta$.

In other words, the concatenation of melodies τ_1 and τ_2 is a melody on the same tier as τ_1 and τ_2, whose event set consists of all events from τ_1 and τ_2, and whose immediate precedence relation consists of all the pairs contained in the corresponding relations of τ_1 and τ_2, plus the condition that the maximal (*i.e.* last) element of τ_1 immediately precedes the minimal (*i.e.*

first) element of τ_2.

From now on, we will represent concatenation using the following standard notation: $\tau = \tau_1 + \tau_2$. Note that this operation is associative; *i.e.* $(\tau_1 + \tau_2) + \tau_3 = \tau_1 + (\tau_2 + \tau_3)$, and therefore can be regarded as having the 'flat' structure $\tau_1 + \tau_2 + \tau_3$. In addition, concatenation is consistency-preserving; *i.e.* if τ_1 and τ_2 are both consistent then their concatenation is also consistent.

3.2 Autosegmental Representations

Phonological representations of the sort used in autosegmental phonology typically consist of a number of tiers, where association can occur between pairs of units (segments, autosegments, X-slots etc.) on different tiers. This situation will be represented using *event structures*, which can be defined as follows (see van Benthem 1983, 1985 for detailed discussions):

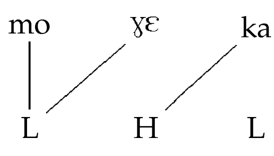
(25) An *event structure* is an ordered triple $\langle E, \prec, \circ \rangle$, where E is a set of events, and $\prec$ and $\circ$ are sets of precedence and overlap relations defined on pairs of elements from E and satisfying the axioms in (13).

However, we will actually require a slightly more complicated kind of event structure in order to represent tiers. In place of the set E of events, we require a set $\mathcal{T}$ of melodies. The corresponding structure will be called a *phonological event structure*.

(26) A *phonological event structure* is an ordered triple $\mathcal{R} = \langle \mathcal{T}, \prec, \circ \rangle$ composed of a set of (consistent) melodies $\mathcal{T} = \{\tau_1, \ldots, \tau_n\}$ where each τ_i has a distinct type, and two sets of ordered pairs specifying the precedence and overlap constraints existing between elements of distinct melodies from $\mathcal{T}$.

For convenience, the term *event structure* will be used hereafter to refer to a *phonological event structure*.

This definition can be best understood by way of a simple example. Consider the autosegmental representation in (27), taken from (Clements 1984:285):

(27)


This representation consists of two tiers, the upper one containing tone-bearing units and the lower one containing tones. In the formalism, we have $\mathcal{R} = \langle \{\tau_{\mathbf{u}}, \tau_{\mathbf{t}}\}, \prec, \circ \rangle$, where $\tau_{\mathbf{u}}$ and $\tau_{\mathbf{t}}$ are melodies on the tone-bearing unit tier and tone tier respectively, and where $\prec$ and $\circ$ determine the relations of overlap and precedence holding between events on these two tiers. Thus, corresponding to the autosegmental phonology notion of a tone tier, we postulate a *type* (in this case, **t**), and corresponding to the *contents* of such a tier, there is a *melody* (in this case, $[\mathrm{L}, \mathrm{H}, \mathrm{L}\,]_{\mathbf{t}}$).

A fuller formalization of (27) is given in (28) :

(28) $\langle \{ [\iota_1{:}\mathbf{mo}, \iota_2{:}\mathbf{ge}, \iota_3{:}\mathbf{ka}\,]_{\mathbf{u}}, [\iota_4{:}\mathbf{L}, \iota_5{:}\mathbf{H}, \iota_6{:}\mathbf{L}\,]_{\mathbf{t}}\ \}, \emptyset\ , \{ \langle \iota_1, \iota_4 \rangle, \langle \iota_2, \iota_4 \rangle, \langle \iota_3, \iota_5 \rangle \} \rangle$

It will be seen that $\prec$ is empty, and thus establishes no precedence constraints across the two melodies. By contrast, $\circ$ imposes a variety of overlap constraints, dictating for example that the tone-bearing units **mo** and **ge** both overlap (are associated with) the first low tone event **L**.

As an aid to reading such event structures, it is often useful to revert to the standard infix notation for relational statements. The clauses which are used to describe overlap and precedence within an event structure will be referred to generically as *constraints*. Following on from this, examples like (28) can be abbreviated to (29), which has the form $\langle \tau_1, \ldots, \tau_n\ ; c_1, \ldots, c_k \rangle$, consisting of a collection of melodies and a collection of constraints. Where possible, the constraints are chained together, so we can write $\iota_1 \circ \iota_4 \circ \iota_2$ rather than $\iota_1 \circ \iota_4, \iota_4 \circ \iota_2$.

(29) $\langle [\mathbf{mo}, \mathbf{ge}, \mathbf{ka}\,]_{\mathbf{u}}, [\mathbf{L_1}, \mathbf{H}, \mathbf{L_2}\,]\ ; \mathbf{mo} \circ \mathbf{L_1} \circ \mathbf{ge}, \mathbf{ka} \circ \mathbf{H} \rangle$

3.3 Representing Hierarchy

At the very beginning of this section we said that the property π associated with an event could itself be a collection of events and constraints. That is, given an event $\iota : \pi$, we allow π to not only be a basic property (such as the tone **H**), but also an event structure $\mathcal{R}$. Suppose, for example, that we wish to describe a phonological phrase α consisting of two phonological words. At the top level, this might be represented as the structure $\mathcal{R} = \langle \{ \omega_1 : \pi_1, \omega_2 : \pi_2 \}\ ; \omega_1 \prec \omega_2 \} \rangle$ However, we will want to provide further structure to both π_1 and π_2. Indeed, we might take the first word ω_1 to be that treated in (29), so that we can further specify $\mathcal{R}$ as (30):

(30) $\langle \{ \omega_1{:}\ \langle [\mathbf{mo}, \mathbf{ge}, \mathbf{ka}\,]_{\mathbf{u}}, [\mathbf{L_1}, \mathbf{H}, \mathbf{L_2}\,]\ ; \mathbf{mo} \circ \mathbf{H_1} \circ \mathbf{ge}, \mathbf{ka} \circ \mathbf{L} \rangle,$
$\omega_2{:}\ \langle \tau_1, \ldots, \tau_n\ ; c_1, \ldots, c_k \rangle \}\ ; \omega_1 \prec \omega_2 \} \rangle$

Given this conception of recursive structure, we can define a notion of hierarchy that will be used to represent multi-tiered, hierarchical phonological structures. The hierarchy is summarised in Figure 1. The relationship between events on different levels of the diagram in Figure 1 will be called *dominance*, an irreflexive, asymmetric and transitive relation. As we showed in §1, simply stating the properties of a relation is not enough; it is also necessary to state how it interacts with the other relations.

First let us consider the relationship between dominance and precedence. Example (31) illustrates two hierarchical units of some kind (the details are not important), both of which dominate specifications for the **nasal** and **continuant** features. The arrows represent the dominance relation, and they are dotted to indicate the possibility of intervening levels of hierarchy (as is found, for example, in Clements 1985:248).

(31) [EXAMPLE NOT AVAILABLE]

We presume such a diagram is to be interpreted as expressing at least the following three precedence constraints: **a** $\prec$ **b**, **+nasal** $\prec$ **–nasal** and **–cont** $\prec$ **+cont**. The number of constraints increases with the amount of internal structure ascribed to **a** and **b**. It is possible to enable greater economy of expression if precedence constraints can be *inherited* up and down the hierarchy. Indeed, inheritance is one of the motivations for adopting hierarchies in the first place.

There appear to be two alternatives here. The first is that **a** $\prec$ **b** holds if and only if *everything* that is dominated by **a** precedes *everything* dominated by **b**. However this would enforce a rigid segmentalism, prohibiting (in combination with axiom (13d) the possibility of **–nasal** overlapping **–cont** , which arguably occurs when an intrusive stop is present (see §3.4). Instead we adopt the following as a working hypothesis:

(32) **Inheritance of Precedence:** Suppose that ϵ_1 and ϵ_2 are two events of the same type, and that each event dominates a number of melodies. For any such melody τ_θ, every event in the event set of τ_θ will also be of type θ (by assumption of consistency). For each type θ of melody in ϵ_1 and ϵ_2, $\epsilon_1 \prec \epsilon_2$ iff all the events of type θ that ϵ_1 dominates precede all the events of type θ that ϵ_2 dominates. The converse is also true (see the Appendix for more detail).

In other words, precedence is inherited up and down the hierarchy, *but only between events of the same type*. For instance, in (32), we gain the result that **a** $\prec$ **b** iff both **+nasal** $\prec$ **–nasal** and **–cont** $\prec$ **+cont**. Crucially, however, **a** $\prec$ **b** does not imply any precedence relation between **+nasal** and **+cont**, or between **–cont** and **–nasal** .

Next we come to consider the relationship between dominance and overlap. From the above discussion, along with axiom (13d), dominance already has an effect on overlap: pairs of events to which the precedence relation is inherited must not overlap. Beyond this very weak interaction between the two relations we are not aware of any others that are necessary.

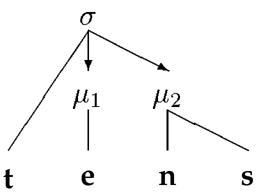
It remains to be explicit about how types are inherited, and to generalize the definition of concatenation already given. These will only be given informally here, with the full definitions in the official notation stated in the Appendix. The most important claim is that the type assigned to an event structure is a 'compound' type, analysed as the *set* of types of the melodies it is made up of. So, for instance, the type of an event structure containing melodies of types of **t** and **u** will be the compound type {**t**, **u**}.

Finally, the concatenation of two event structures $\mathcal{R}_1$ and $\mathcal{R}_2$ of the same type is defined to be the event structure formed by concatenating the component melodies of $\mathcal{R}_1$ and $\mathcal{R}_2$ in a pairwise manner, and by forming the union of the constraint sets.

3.4 An Example

In this section we provide a small illustration of event structures as applied to autosegmental representations. Consider the word **tense**, where we can observe three widespread phenomena, namely aspiration of the initial voiceless stop, regressive nasalization and an intrusive stop. This word is a monosyllable, so at the syllable level of prosodic hierarchy we can posit an event ϵ_σ, where the subscript indicates the type. This event, like all events, consists of an interval and a property. The property is complex; it is itself an event structure consisting of moras. Both the syllable and the mora events can be associated to segments (Hyman 1984, 1985, Hayes 1989). (33) is a syllable structure diagram for the word **tense**, where the solid-headed arrows represent dominance and the lines represent association:

(33)



The syllable event ϵ_σ can accordingly be expanded to $\iota_1 : [\iota_2{:}\mu_1,\ \iota_3{:}\mu_2\ ]_\sigma$, and this structure can then be linked to the segments $[\mathbf{t}, \mathbf{e}, \mathbf{n}, \mathbf{s}\ ]_\mathrm{s}$ to get (34a), abbreviated as (34b).

(34) a. $\langle\, \iota_1{:}[\iota_2{:}\mu_1,\ \iota_3{:}\mu_2\ ]_\sigma,\ [\iota_4{:}\mathbf{t},\ \iota_5{:}\mathbf{e},\ \iota_6{:}\mathbf{n},\ \iota_7{:}\mathbf{s}\ ]_\mathrm{s}\ ;\ \iota_1 \circ \iota_4,\ \iota_2 \circ \iota_5,\ \iota_3 \circ \iota_6,\ \iota_3 \circ \iota_7\ \rangle_{\sigma,\mathrm{s}}$

b. $\langle\, \iota_1{:}[\mu_1, \mu_2\ ]_\sigma,\ [\mathbf{t}, \mathbf{e}, \mathbf{n}, \mathbf{s}\ ]_\mathrm{s}\ ;\ \iota_1 \circ \mathbf{t},\ \mu_1 \circ \mathbf{e},\ \mu_2 \circ \mathbf{n},\ \mu_2 \circ \mathbf{s}\ \rangle$

(The type **s** indicates a segmental tier, and the type σ is, strictly speaking, an abbreviation for the compound type $\{\mu\}$ based on the type of moras; see the Appendix for details.)

A partial description of the articulator motions involved in an utterance of the word **tense** appears in Figure 2; it makes use of the conventions proposed by Browman & Goldstein (1989). This expresses the claim, for example, that the aperture at the velum (VEL) is wide for the articulation of **n**. The other rows correspond to parameters for the tongue-body, tongue-tip and glottis respectively. The delay in onset of voicing after the release of the alveolar closure is the reason for the observed aspiration of **t**, the widening of the velic opening prior to alveolar closure causes the observed partial nasalization on the vowel, and the lowering of the velum before the release of the alveolar closure gives rise to the intrusive stop (cf. Clements 1987 for a treatment of intrusive stop formation using feature geometry).

From a phonological perspective these properties are non-contrastive, and so do not belong in a phonological representation. However, rather than deriving a range of forms (e.g. [$\mathrm{t^h\tilde{e}n^ts}$])

from supposedly more basic forms (e.g. [tens]) using optional insertion rules, an appropriate event structure can *admit* all of these forms without assigning priority to any one form derivationally. We replace the segments **t**, **e**, **n** and **s** in (34) with event structures which refer explicitly to articulatory gestures, as specified in (35). (Note that the type θ of each event structure in (35) is { VEL,TB,TT,GLO }, even though each event structure only contributes melodies to a subset of these types—this is a technicality necessitated by our definition of concatenation.)

(35)

$$\begin{array}{lcl} \mathbf{t} & = & \langle [\iota_1\text{: }\mathbf{(closure, alveolar)}\ ]_{\mathbf{TT}}, [\iota_2\text{: }\mathbf{wide}\ ]_{\mathbf{GLO}}\ ; \iota_1 \circ \iota_2 \rangle_\theta \\ \mathbf{e} & = & \langle [\iota_3\text{: }\mathbf{(mid, palatal)}\ ]_{\mathbf{TB}} \rangle_\theta \\ \mathbf{n} & = & \langle [\iota_4\text{: }\mathbf{wide}\ ]_{\mathbf{VEL}}, [\iota_5\text{: }\mathbf{(closure, alveolar)}\ ]_{\mathbf{TT}}\ ; \iota_4 \circ \iota_5 \rangle_\theta \\ \mathbf{s} & = & \langle [\iota_6\text{: }\mathbf{(critical, alveolar)}\ ]_{\mathbf{TT}}, [\iota_7\text{: }\mathbf{wide}\ ]_{\mathbf{GLO}}\ ; \iota_6 \circ \iota_7 \rangle_\theta \end{array}$$

In (35), following (Browman & Goldstein 1989), all events are linked into the same level of prosodic constituent structure (the root tier). An alternative would be for some events, such as those involving the glottis, to be associated to higher-level units of prosodic structure (such as onsets or moras). The formalism as presented makes no commitment either way; our concern here is simply to provide a detailed illustration.

There are at least two ways to achieve the explicit relative timings of the two gestures involved in the articulation of the segment **t** as shown in Figure 2—namely that the alveolar closure begins before the widening of the glottal opening begins, and that the closure ends before the widening of the glottal opening ends. We could extend the overlap statements, such as $\epsilon_1 \circ \epsilon_2$, by selecting one of the options in Figure 3, where each line represents the interval occupied by an event, and where the top line of each pair represents ϵ_1 and the bottom line represents ϵ_2. In this case we would select option 9. However this involves reference to endpoints, and we have already indicated some of our reservations about relying on these. Another option, and the one we favour, is to give events finer sub-structure (e.g. a **[wide]** event consists of an **[opening]** event followed by a **[maximally-open]** event followed in turn by a **[closing]** event), and then to require the overlap of the opening and maximally-open events with the alveolar closure event and prohibit (see §4) the overlap of the closing event with the alveolar closure.

3.5 The Segment

From the above it can be seen that a revised notion of the segment has been called into play. The traditional notion of segment can be expressed in our notation as follows. Consider an event structure for the segment **[+F,–G,+H]** (where we assume that distinctive features correspond to types) :

(36) $\langle\ [\iota_1\text{:}+]_{\mathbf{F}}\ ; [\iota_1\text{:}-]_{\mathbf{G}}\ ; [\iota_1\text{:}+]_{\mathbf{H}}\ \rangle$

Each of the three events consist of the *same* interval ι_1 along with their individual property. However, such a view of the segment is difficult to defend in the face of phonetic considerations. In her overview of the phonetic basis for segments, Keating (1988:292) adopts the widely-held position that discrete segments are not directly attested in the phonetic signal, and that 'the component features of segments [are] misaligned with each other in time'. This alternative notion of segment can be readily encompassed within our framework; consider the revised event structure for the segment **[+F,–G,+H]** in (37):

(37) $\langle\, [\iota_1\!:\!+]_{\mathbf{F}}\,;[\iota_2\!:\!-]_{\mathbf{G}}\,;[\iota_3\!:\!+]_{\mathbf{H}}\ ;\ \iota_1 \circ \iota_2, \iota_2 \circ \iota_3, \iota_3 \circ \iota_1 \,\rangle$

In contrast with (36), (37) has a distinct interval for each event, and each of these intervals are merely required to *overlap* the others. Thus, although the traditional notion of segment can be modelled as a set of coterminous events, it has no privileged status in our ontology. Consequently we view the segment as epiphenomenal.

What crucially distinguishes our approach from most others is that the relationship between events in general can range from being highly constrained to highly underdetermined, thus affording a degree of flexibility that is typically not found in either segment-based or autosegment-based approaches.

4 Conclusion and Future Prospects

At the outset we mentioned three kinds of interpretation of autosegmental representations: formal, phonetic and computational. In developing the notions of event in §2 and event structure in §3 we have been mainly concerned with formal interpretation. An interesting aspect of this interpretation is its one-dimensionality: the only formally relevant *dimension* in the context of our interpretation of autosegmental representations is the single dimension of time; the geometric dimensions play no part. This is as it should be, for even though there is an attractive analogy between our intuitive conception of sound structure and certain graphical notations, there have been no arguments advanced (as far as we are aware) that a particular property of these graphical notations, namely geometric *dimension*, has any observable exponent. In the absence of such argumentation, to build such a notion into a phonological formalism would be to push the analogy too far.

The example in §3.4 illustrated, amongst other things, the direct but flexible link to a phonetic level of description of the kind advocated by Browman and Goldstein (1989). The phenomenon of epenthetic stop formation in nasal+fricative coda clusters, often expressed using phonological rules (e.g. Clements 1987), could be modelled naturally using event structures. Of course, the presentation of this small example is not intended as an argument that all phonological processes can be treated in this way. To be sure, any attempt to translate a standard autosegmental analysis into one using event structures will have to pay at least as much attention to the formal interpretation of autosegmental *rules* as we have paid to the interpretation of autosegmental representations. A possible interpretation of an autosegmental rule is as a function relating two sets of event structures. A derivation may be interpreted as the composition of such functions. Such rules would be able to refer directly to the information about hierarchy, locality, association and multi-tiered structure of a representation which an event structure provides, but have no reference to, or control over, the exact temporal extent of events.

Another motivating factor behind our interest in formalisation is the view that the rule systems developed in generative phonology are excessively *procedural*. This connects with the familiar debate about the metatheoretical undesirability of extrinsically (or parochially) ordered rules (e.g. Koutsoudas, Sanders and Noll 1974, Pullum 1976), and with earlier complaints that the derivational stance of generative phonology was inherently process-oriented. The issues are still valid, we believe, and a useful new slant can be given by drawing lessons from recent work in computation and constraint-based grammar formalisms.

It is not possible to adequately address such controversial issues within the confines of this paper. Nevertheless, it would be useful to briefly indicate some points of contact between our approach and that adopted by computational and theoretical linguists working outside the spheres of phonology and phonetics.

A rather natural step in the development of the formalism is to view phonological representations as providing *partial information* about linguistic objects. Thus, the sequence of event structures in (38) provide successively more information about an utterance event.

(38) a. $\langle \epsilon_\sigma, \epsilon_{\mathrm{s}} \rangle_{\sigma,\mathrm{s}}$
b. $\langle \iota{:}[\iota{:}\mu, \iota{:}\mu\,]_\sigma, [\iota{:}\mathbf{t}, \iota{:}\mathbf{e}, \iota{:}\mathbf{n}, \iota{:}\mathbf{s}\,]_{\mathrm{s}} \rangle_{\sigma,\mathrm{s}}$
c. $\langle \iota_1{:}[\iota_2{:}\mu, \iota_3{:}\mu\,]_\sigma, [\iota_4{:}\mathbf{t}, \iota_5{:}\mathbf{e}, \iota_6{:}\mathbf{n}, \iota_7{:}\mathbf{s}\,]_{\mathrm{s}}\ ; \iota_1 \circ \iota_4, \iota_2 \circ \iota_5, \iota_3 \circ \iota_6, \iota_3 \circ \iota_7 \rangle_{\sigma,\mathrm{s}}$

In this sequence we can observe the increase in 'information content' — a notion which has been formulated explicitly in certain grammatical formalisms using the linked notions of subsumption and unification (cf. Shieber 1986). The operation of unification (written $\bigsqcup$) takes two partial information structures and yields a third which combines all the information present in the first two . (The operation is associative and so the order in which partial descriptions are combined is immaterial.) Given reasonable assumptions about the merging of information in event structures under unification, we can take (39) to be equivalent to (38c).

(39) $\langle \iota_1{:}[\iota_2{:}\mu, \iota_3{:}\mu\,]_\sigma, [\epsilon_4, \epsilon_5, \epsilon_6, \epsilon_7\,]_{\mathrm{s}}\ ; \iota_1 \circ \epsilon_4, \iota_2 \circ \epsilon_5, \iota_3 \circ \epsilon_6, \iota_3 \circ \epsilon_7 \rangle_{\sigma,\mathrm{s}} \bigsqcup \langle \epsilon_\sigma, [\mathbf{t}, \mathbf{e}, \mathbf{n}, \mathbf{s}\,]_{\mathrm{s}} \rangle_{\sigma,\mathrm{s}}$

Unification fails if there are conflicting specifications in the operands; for example, $\iota{:}\mathbf{t} \bigsqcup \iota{:}\mathbf{n}$ is undefined.

Yet a further step would involve moving away from information structures to *descriptions* of those structures, couched within an appropriate deductive calculus (cf. Johnson 1988 for a detailed exposition of this approach). That is, phonological representations would consist of event descriptions, and having models (i.e. the structures of which such descriptions are true or false) provided by event structures of the kind presented in the body of this paper. One advantage of such a move is that it would provide a straightforward formal basis for adding logical negation and disjunction to our representations; see also Karttunen (1984), Kasper & Rounds (1986), Moshier & Rounds (1987) for general discussion of this issue. Another possible advantage of taking this step is that it would give us a formal framework in which to explore the proposal that the relation between phonology and phonetics is analogous to the distinction between syntax and (model-theoretic) semantics. This distinction is rarely made (if at all) in the phonology literature, but nevertheless is one which we believe to be important.

We believe the formal interpretation of multi-tiered, hierarchical autosegmental representations given here is faithful to the intentions underlying several diagrammatic conventions currently in wide use. Several benefits follow from this. First, autosegmental representations can be described succinctly, without recourse to diagrams or prose, although we do not deny that these latter have their place; indeed the opportunities for visualization offered by an apt graphical notation are invaluable. Second, inference and consistency checking can be performed directly on representations. Third, abbreviatory devices can be stated formally, with the consequence that using abbreviated forms is less likely to lead to confusion. Fourth, theoretical and empirical claims can be made more explicit if their substantive content is unambiguous, allowing a formal comparison of competing analyses to be made. Fifth, the formalism lays the groundwork for a computational representation of autosegmental representations—a development which will enable the automatic checking of the correspondence between an analysis and its target data.

In conclusion, we hope is that autosegmental phonologists will find this formalism, or derivatives of it, useful in situations where a greater degree of precision is required than that standardly afforded by diagrams or prose. We also hope that our proposals will contribute to a similar formalization of other extant approaches to phonology, thereby allowing linguists to obtain a clearer picture of the precise claims made by competing theoretical frameworks.

Footnotes

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1. In fact, these properties only give us a strict partial ordering; to get a linear ordering, we also need an additional statement of connectedness: 'For all x and y, either x precedes y or $x = y$ or y precedes x'. We will return to this once tiers and melodies have been defined. Note that Sagey (1988:110) uses the term 'antisymmetric' when 'asymmetric' is intended (see, for example, Suppes 1972:69 for definitions of these properties).

2. We are indebted to one of the anonymous *JL* referees for suggesting the use of these English paraphrases. The axioms in (13) and the inference rule in (14) are intended to be couched in a theory of classical first-order logic. We refrain from presenting the proof theory here.

3. It has often been observed that what we have called the homogeneous/heterogeneous distinction for events has a parallel in mass/count distinction for objects (e.g. Taylor 1977:210-11).

4. To see why this is so, consider four intervals x_1, x_2, x_3 and ϵ, where $x_1 \prec x_2$, $x_2 \prec x_3$, $x_1 \circ \epsilon$, and $x_3 \circ \epsilon$. From (11b) we know there are points $p_1 \in x_1$ and $q_1 \in \epsilon$ such that $p_1 = q_1$. Now from (11a), for any $p_2 \in x_2$, $q_1 < p_2$. By a similar argument, for some $q_3 \in x_3$, $p_2 < q_3$. Therefore $p_2 \in \epsilon$, and so $x_2 \circ \epsilon$.

References

Batóg, Tadeusz. (1967). *The Axiomatic Method in Phonology.* London: Routledge & Kegan Paul.

Bennett, M. & Barbara Partee. (1972). *Toward the Logic of Tense and Aspect in English.* Indiana University Linguistics Club.

Benthem, Johan van. (1983). *The Logic of Time – A Model-Theoretic Investigation into the Varieties of Temporal Ontology and Temporal Discourse.* Synthese Library, Volume 156. Dordrecht: Reidel.

Benthem, Johan van. (1985). *A Manual of Intensional Logic.* Second Edition. Center for the Study of Language and Information, Lecture Notes Number 1.

Bloch, Bernard. (1948). A set of postulates for phonemic analysis. *Language.* **24.** 3-46.

Bloomfield, Leonard. (1926). A set of postulates for the science of language. *Language.* **2.** 153–164. Reprinted in Martin Joos (ed.) *Readings in Linguistics I: The Development of Descriptive Linguistics in America 1925–56.* 26–31.

Browman, Catherine & Louis Goldstein. (1986). Towards an articulatory phonology. In Colin Ewen & John Anderson (eds.) *Phonology Yearbook.* **3.** 219–252.

Browman, Catherine & Louis Goldstein. (1989). Articulatory gestures as phonological units. *Phonology* **6**(2).

Clements, George N. (1984). Principles of Tone Assignment in Kikuyu. In George N. Clements & John Goldsmith (eds.) *Autosegmental Studies in Bantu Tone.* 281–339. Dordrecht: Foris.

Clements, George N. (1985). The geometry of phonological features. In Colin Ewen & John Anderson (eds.) *Phonology Yearbook.* **2.** 225–252.

Clements, George N. (1987). Phonological features representation and the description of intrusive stops. In Anna Bosch, Barbara Need & Eric Schiller (eds.) *Papers from the 23rd Annual Regional Meeting of the Chicago Linguistic Society.* Part Two: Parasession on Autosegmental and Metrical Phonology. Published by the Chicago Linguistic Society.

Ewen, Colin J. (1986). Segmental and suprasegmental structure. In J. Durand (ed.) *Dependency and Non-linear Phonology.* London: Croom Helm.

Goldsmith, John. (1976). *Autosegmental Phonology.* Doctoral Thesis, Massachusetts Institute of Technology. Garland Publishing, Inc. (1979).

Greenberg, J.H. (1959). An axiomatization of the phonologic aspect of language. In L. Gross (ed.) *Symposium on Sociological Theory.* 437–480. New York: Evanston.

Hammond, Michael. (1988). On deriving the well-formedness condition. *Linguistic Inquiry* **19**(2). 319–325.

Hayes, Bruce. (1989). Compensatory Lengthening in Moraic Phonology. *Linguistic Inquiry* **20**(2). 253–306.

Hoberman, Robert D. (1988). Local and Long-Distance Spreading in Semitic Morphology. *Natural Language and Linguistic Theory.* **6**(4). 541–549.

Hyman, Larry M. (1984). On the weightlessness of syllable onsets. In C. Brugman & M. Macaulay (eds.) *Proceedings of the Tenth Annual Meeting of the Berkeley Linguistics Society*. University of California, Berkeley.

Hyman, Larry M. (1985). *A theory of phonological weight.* Publications in Language Sciences 19. Dordrecht: Foris Publications.

Johnson, Mark. (1988). *Attribute-Value Logic and the Theory of Grammar.* Center for the Study of Language and Information, Lecture Notes Number 16.

Karttunen, Lauri. (1984). Features and values. *Tenth International Conference on Computational Linguistics.* 28–33.

Kasper, Robert T. & William C. Rounds. (1986). A logical semantics for feature structures. *24th Annual Meeting of the Association for Computational Linguistics.* 257–266.

Keating, Patricia A. (1989). The phonology-phonetics interface. In Frederick J. Newmeyer (ed.) *Linguistics: The Cambridge Survey*. Volume 1: 281–302.

Koutsoudas, Andreas, Gerald Sanders & Craig Noll. (1974). The application of phonological rules. *Language.* **50.** 1–28.

Lass, Roger. (1983). *Phonology – An Introduction to Basic Concepts*. Cambridge University Press.

Moshier, M. D. and W. C. Rounds. (1987). A logic for partially specified data structures. *ACM Symposium on the Principles of Programming Languages*. Association for Computing Machinery.

Pierrehumbert, Janet & Mary Beckman. (1988). *Japanese Tone Structure.* Linguistic Inquiry Monograph Fifteen. Massachusetts Institute of Technology Press.

Pullum, Geoffrey K. (1976). *Rule Interaction and the Organization of a Grammar.* Garland Publishing, Inc.

Sagey, Elisabeth. (1986). *The Representation of Features and Relations in Non-Linear Phonology.* Doctoral dissertation. Massachusetts Institute of Technology.

Sagey, Elisabeth. (1988). On the ill-formedness of crossing association lines. *Linguistic Inquiry.* **19**(1). 109–118.

Shieber, Stuart. (1986). *An Introduction to Unification-Based Approaches to Grammar.* Center for the Study of Language and Information, Lecture Notes Number 4.

Suppes, Patrick. (1972). *Axiomatic Set Theory.* New York: Dover Publications, Inc.

Taylor, Barry. (1977). Tense and Continuity. *Linguistics and Philosophy.* **1.** 199–220.

Appendix: Summary of Formalism

Let Θ be a set of *basic types*. A *type* is either a basic type, or a set of basic types, or a set of sets of basic types, etcetera. For each element θ of Θ there is a set of *basic values* Π_θ. A *basic property* π_θ is an element π of Π_θ, indexed with its type θ.

An *event* is an ordered pair $\langle \iota, \pi_\theta \rangle_\theta$ where ι is an interval and where π_θ is either a basic property (in which case we say the event is *atomic*), a melody or a phonological event structure (as defined below). Note that the type of an event is the same as the type of the property of that event.

Recall that a *totally ordered set* is an ordered pair $\langle S, \prec \rangle$, where S is a set and $\prec \subset S \times S$ is an irreflexive, asymmetric, transitive and linear relation.

A *melody* τ is an ordered pair $\langle E, \prec^\circ \rangle_{\{\theta\}}$, where E consists of events of type θ and $\prec^\circ \subset E \times E$. The relation $\prec^\circ$ is irreflexive, asymmetric and intransitive, and has a unique embedding in a set $\prec$, such that $\langle E, \prec \rangle$ is a totally ordered set. E is called the *event set* of τ. An element e of E is $\prec$*-maximal* iff for all $e_1 \in E$, either $e_1 \prec e$ or $e = e_1$, and analogously for $\prec$*-minimal*. A melody τ has unique maximum and minimum elements with respect to $\prec$, denoted $max(\tau)$ and $min(\tau)$ respectively. We often abbreviate $\langle \{e_1, \ldots, e_n\}, \prec^\circ \rangle_{\{\theta\}}$ to $[e_1, \ldots, e_n]_\theta$ where $e_1, \ldots, e_n$ appear in order of precedence, where $min(\tau) = e_1$ and $max(\tau) = e_n$.

The *concatenation* of two melodies τ_1 and τ_2 of the same type θ, where $\tau_1 = \langle E_1, \prec_1^\circ \rangle_\theta$ and $\tau_2 = \langle E_2, \prec_2^\circ \rangle_\theta$, is a third melody of type θ, namely $\tau_3 = \langle E_1 \cup E_2, \prec_1^\circ \cup \prec_2^\circ \cup \{\langle max(\tau_1), min(\tau_2) \rangle\} \rangle_\theta$. We write $\tau_1 + \tau_2 = \tau_3$.

A *phonological event structure* $\mathcal{R}$ (generally referred to as an *event structure* in the body of this paper for convenience) is an ordered triple
$\langle \{\tau_{\theta_1}, \ldots, \tau_{\theta_n}\}, \prec, \circ \rangle_{\{\theta_1, \ldots, \theta_n\}}$ where $\prec, \circ \subset \bigcup_{i \neq j} E_i \times E_j$, where $\prec$ and $\circ$ satisfy A1–A5 below, and where each E_i corresponds to the event set of τ_{θ_i}.

The melodies in an event structure have distinct types—*i.e.* $\theta_i = \theta_j$ implies $i = j$. The abbreviated notation for event structures is: $\langle \tau_{\theta_1}, \ldots, \tau_{\theta_n} ; c_1, \ldots, c_m \rangle_{\theta_1, \ldots, \theta_n}$, where the c_i are pairs from the sets $\prec$ and $\circ$ written with the relation appearing between its arguments.

It will frequently be the case that a melody in an event structure is empty. According to the above convention, this is represented thus: $[\,]_{\theta_i}$. Instead of cluttering the representations unnecessarily, we prefer to omit such melodies, and instead to signal the existence of an empty melody by entering its type in the type subscript of the event structure.

Event Axioms

A1:	$\forall x$	$x \circ x$
A2:	$\forall xy$	$x \circ y \rightarrow y \circ x$
A3:	$\forall xy$	$x \prec y \rightarrow \neg y \prec x$
A4:	$\forall xy$	$x \prec y \rightarrow \neg x \circ y$
A5:	$\forall wxyz$	$w \prec x \wedge x \circ y \wedge y \prec z \rightarrow w \prec z$

Note that C1–C3 below are direct consequences of these axioms:

C1:	$\forall x$	$\neg x \prec x$
C2:	$\forall xyz$	$x \prec y \wedge y \prec z \rightarrow x \prec z$
C3:	$\forall xy$	$x \circ y \rightarrow \neg x \prec y \wedge \neg x \succ y$

C1 follows from A3 by setting $x = y$ and deriving a contradiction, C2 follows from A1 and A5, setting $x = y$ in A5, C3 and A4 are equivalent.

A third relation is induced by the hierarchy of event structures; we call it *dominance* and it is defined as follows:

An event e dominates a melody τ, written $e\ \delta\ \tau$ iff

(i) $e = \iota{:}\langle \mathcal{T}, \prec, \circ \rangle$, where $\tau \in \mathcal{T}$, or

(ii) $e = \iota{:}\tau$, or

(iii) $e\ \delta\ e'$ and $e'\ \delta\ \tau$

An event e dominates another event e', written $e\ \delta\ e'$ iff $e\ \delta\ \tau$, where $\tau = \langle E, \prec^{\circ} \rangle$ and $e' \in E$.

Dominance interacts with precedence as follows. If e_1 and e_2 are complex events of type θ, then $e_1 \prec e_2$ iff $\forall \theta' \in \theta\ \forall e_3, e_4$ such that $e_1\ \delta\ e_3$ and $e_2\ \delta\ e_4$, $e_3 \prec e_4$, where e_3 and e_4 both have type θ'.

The *concatenation* of two event structures is defined as follows:
$\langle \{ \tau_{\theta_1}, \ldots, \tau_{\theta_n} \}, \prec_1, \circ_1 \rangle_{\{\theta_1 \ldots \theta_n\}} + \langle \{ \tau'_{\theta_1}, \ldots, \tau'_{\theta_n} \}, \prec_2, \circ_2 \rangle_{\{\theta_1 \ldots \theta_n\}}$
$= \langle \{ \tau_{\theta_1} + \tau'_{\theta_1}, \ldots, \tau_{\theta_n} + \tau'_{\theta_n} \}, \prec_1 \cup \prec_2, \circ_1 \cup \circ_2 \rangle_{\{\theta_1 \ldots \theta_n\}}$.

Figure 1: The Prosodic Hierarchy Using Recursive Event Structures.

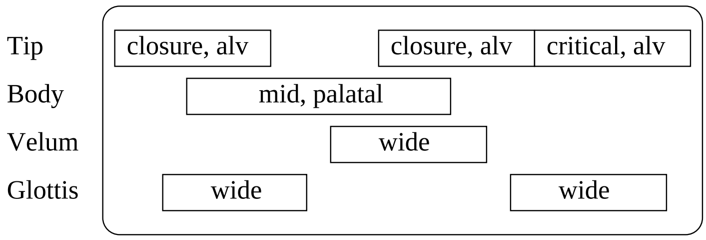

Figure 2: Gestural Score for ‘tense’.

Figure 3: Options for the Overlap of Intervals.