Stabilization of the Earth's obliquity by the Moon

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ACCORDING to Milankovitch theory[1,2], the ice ages are related to variations of insolation in northern latitudes resulting from changes in the Earth's orbital and orientation parameters (precession, eccentricity and obliquity). Here we investigate the stability of the Earth's orientation for all possible values of the initial obliquity, by integrating the equations of precession of the Earth. We find a large chaotic zone which extends from 60° to 90° in obliquity. In its present state, the Earth avoids this chaotic zone and its obliquity is essentially stable, exhibiting only small variations of ±1.3° around the mean value of 23.3°. But if the Moon were not present, the torque exerted on the Earth would be smaller, and the chaotic zone would then extend from nearly 0° up to about 85°. Thus, had the planet not acquired the Moon, large variations in obliquity resulting from its chaotic behaviour might have driven dramatic changes in climate. In this sense one might consider the Moon to act as a potential climate regulator for the Earth.

Ward[3] suggested that, in the absence of the Moon and on the basis of its present spin rate, the Earth might have exhibited quasiperiodic variations of about ±10° in obliquity. But he also suggested that without the Moon, the Earth would have had a faster spin rate, leading to greater rotational flattening which would have compensated for the lack of a lunar torque and reduced the obliquity variations to their present value. This conclusion requires that the primordial rotation period rate of the Earth should be less than 8 h. We find that for a slower primordial spin, and over a wide range of initial values, the obliquity variations will be chaotic, with variations much larger than predicted by Ward.

Our model and method of analysis are described in Boxes 1 and 2. We investigate the dynamics of the Earth's obliquity ε ($=\cos^{-1} X$ in Box 1) by integrating the equations of precession ((1) and (2)) over 18 Myr for all values of the initial obliquity ε_0, in steps of 0.1°, from 0° to 170°. The perturbation effect of the Solar System is taken into account by using the secular La90 solution for the Earth[4]. We used the frequency analysis method[4-6] for the analysis of the obliquity and precession (Box 2). Briefly, this method consists of defining for each orbit a frequency vector by a refined Fourier analysis over a finite time span. The regularity of the orbits can then be analysed very precisely by studying the regularity of the frequency application which links the action-like variable (ε) to the frequencies. In Fig. 2*a*, the precession frequency p_f is plotted against the initial obliquity ε_0, for a fixed value of the initial precession angle ψ_0. From $\varepsilon_0=0$ to $\varepsilon_0=60°$, the frequency curve is very regular, and reflects the regular behaviour of the solution. A large chaotic zone then extends from 60° to 90°. For higher values of obliquity, the precession frequency becomes negative; the effects of possible resonances are much smaller, and the motion is again very regular. In Fig. 2*b*, for each integration, the minimum, maximum and mean values of the obliquity reached during the 18 Myr of the integration are given. If, for example, the obliquity goes to 60°, then in a few million years it can reach 90°, because of the planetary secular perturbations alone.

In earlier calculations[7], we suppressed the Moon, keeping the present values for the Earth's parameters. Because the torque exerted on the Earth is reduced ($c_1=c_2=c_3=0$), the precession frequency goes down to $\sim 15.6''\,\mathrm{yr}^{-1}$, which is close to the value of the leading frequencies of $A(t)+iB(t)$ but with the opposite sign (Box 2). As predicted by Ward[3], this led to large variations of the obliquity (from 15° to 32° in 1 Myr).

BOX 1 Equations of precession

The general precession in longitude, ψ, and the obliquity at a given time, ε, are determined by the motions of the equatorial and ecliptic pole. The precession equations are written in terms of the action variable $X=\cos\varepsilon$ and the associated angle variable ψ. Let us denote two new variables by $p=\sin(i^*/2)\sin(\Omega)$, $q=\sin(i^*/2)\cos(\Omega)$ where i^* is the inclination of the Earth with respect to a fixed ecliptic, and Ω the longitude of the node. The equations of precession[7,9,10] can be written

$$\frac{d\psi}{dt}=T(X,t)-\frac{X}{\sqrt{1-X^2}}(A(t)\sin\psi+B(t)\cos\psi) \tag{1}$$

$$\frac{dX}{dt}=-\sqrt{1-X^2}\,(B(t)\sin\psi+A(t)\cos\psi) \tag{2}$$

with

$$T(X,t)=c_1X+c_2(2X^2-1)/(1-X^2)$$
$$+c_3(6X^2-1)+c_4S_0X-2C(t)-p_g$$

and

$$A(t)=2(\dot q+p(q\dot p-p\dot q))/\sqrt{1-p^2-q^2}$$
$$B(t)=2(\dot p-q(q\dot p-p\dot q))/\sqrt{1-p^2-q^2}$$
$$C(t)=(q\dot p-p\dot q)$$

The coefficients c_1 to c_4 and the geodetic precession p_g depend on the orbital parameters of the Moon and the Sun ($c_1=37.526603''\,\mathrm{yr}^{-1}$, $c_2=-0.001565''\,\mathrm{yr}^{-1}$, $c_3=0.000083''\,\mathrm{yr}^{-1}$, $c_4=34.818618''\,\mathrm{yr}^{-1}$, $p_g=0.019188''\,\mathrm{yr}^{-1}$). We also have $S_0=\frac{1}{2}(1-e^2)^{-3/2}-0.522\times10^{-6}$, where e is the eccentricity of the Earth[10]. The initial conditions for the Earth are $\dot\psi(t=0)=50.290966''\,\mathrm{yr}^{-1}$, $\varepsilon(t=0)=23°\,26'\,21.448''$. These equations can be associated with the hamiltonian depending on time

$$H(X,\psi,t)=\mathcal{T}(X,t)+\sqrt{1-X^2}\,(A(t)\sin\psi+B(t)\cos\psi) \tag{3}$$

where

$$\mathcal{T}(X,t)=\tfrac{1}{2}(c_1+S_0c_4)X^2-c_2X\sqrt{1-X^2}$$
$$+c_3(2X^3-X)-(2C(t)+p_g)X$$

To understand the dynamics of this hamiltonian, its small terms (c_2, c_3, p_g, $C(t)$) can be neglected, although they will be taken into account during the numerical computations. We shall also neglect the eccentricity of the Earth and the Moon, as well as the inclination of the Moon. In this case, $c_1=(C-A)/C\times3n_M^2m_M/2\nu$, $S_0=\frac{1}{2}$, $c_4=(C-A)/C\times3n_\odot^2M_\odot/\nu$, where ν is the angular velocity of the Earth, $(C-A)/C$ its dynamical ellipticity (which is proportional to ν^2), n_M and m_M, the mean motion and mass of the Moon, and $n_\odot$ and $M_\odot$ the same quantitites for the Sun. The hamiltonian then reduces to

$$H(X,\psi,t)=\tfrac{1}{2}\alpha X^2+\sqrt{1-X^2}\,(A(t)\sin\psi+B(t)\cos\psi) \tag{4}$$

with $\alpha=c_1+S_0c_4$. The expression $A(t)+iB(t)$ (as well as the eccentricity e of the orbit of the Earth) will be given by the La90 solution of Laskar[4].

Here we look at the possible dynamics of the very early Earth, on the hypothesis that the Moon was not present at the time. If so, because of tidal dissipation, the rotation of the Earth would have been faster, as shown by geological records[8] which give $\nu\approx1.22\,\nu_p$ for -2.5 Gyr (where ν_p is the present rotation velocity of the Earth). We analysed the obliquity variations for this rate of rotation (Fig. 3). In this case, the chaotic region becomes very large, extending from 0° to about 80°. Even if the initial obliquity is small, it can exceed 50° in a few million years. Furthermore, the frequency analysis shows the possibility of diffusion up to about 80°, although this diffusion may be slow.

For a higher rotation speed of $\nu=1.6\,\nu_p$, which may have existed at -4.5 Gyr, a large resonant zone appears, corresponding to the node secular mean rate of Jupiter and Saturn ($s_6=-26.3302''\,\mathrm{yr}^{-1}$) (Fig. 4). For zero initial obliquity, we find variations of $\sim10°$, but as soon as the initial obliquity reaches 4°, the motion can enter the chaotic zone surrounding the resonant island, and show variations of more than 30° in a few million years, with the possibility of further diffusion up to more than 85°.

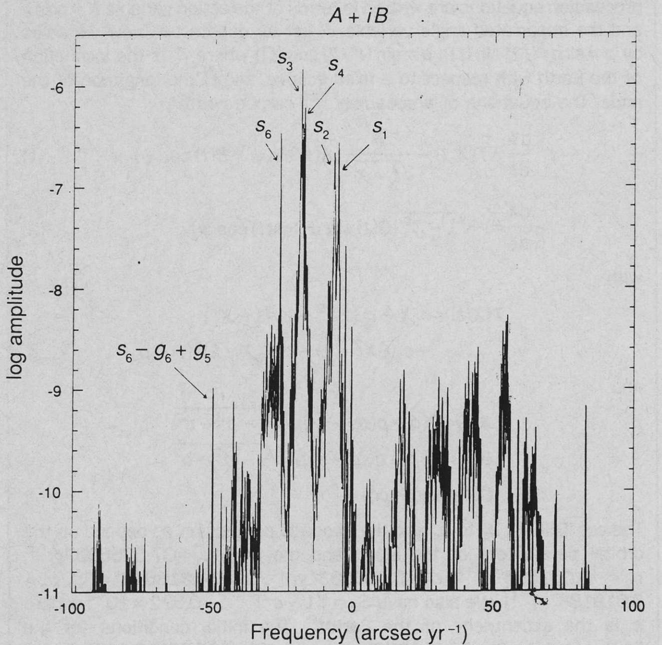

FIG. 1 Fourier spectrum of $A(t)+iB(t)$ over 17 Myr. The main secular frequencies of the Solar System can be identified, as well as the small isolated term $s_6-g_6+g_5$.

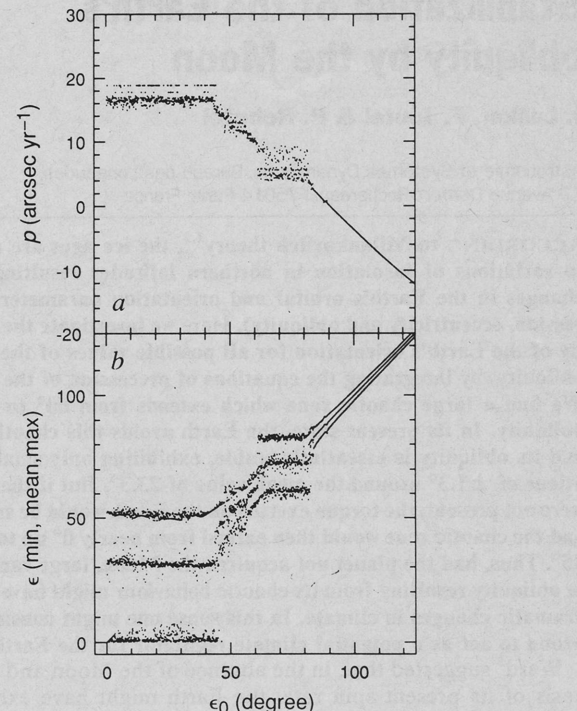

FIG. 3 Without the Moon, the chaotic zone revealed by the frequency analysis over 18 Myr (a) extends from 0° to ~85°, for a rotation velocity of the Earth $\nu=1.22\ \nu_p$. b, Maximum, mean and minimum obliquity reached during 18 Myr.

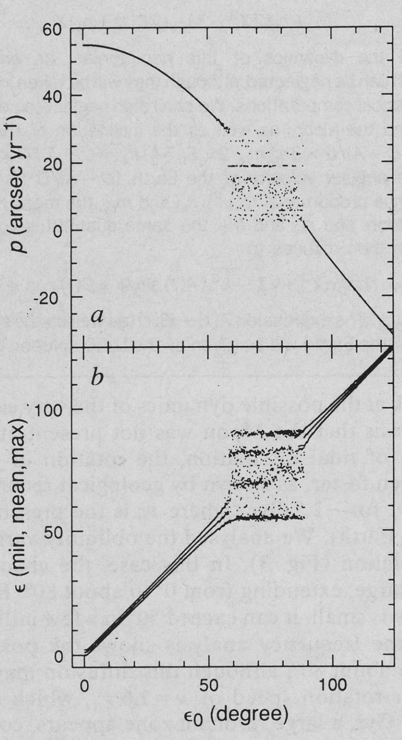

FIG. 2 Frequency analysis over 18 Myr of the precession of the Earth under lunar and solar torque for all values of the initial obliquity (ε_0). a, A large chaotic zone exists from about 60° to 90°. b, Maximum, mean and minimum obliquity reached during 18 Myr.

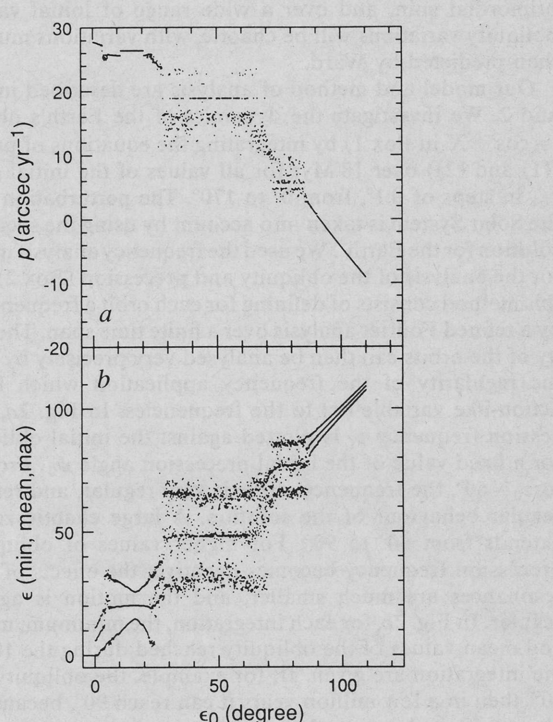

FIG. 4 For $\nu=1.6\ \nu_p$, the large chaotic zone extends from nearly 0° to ~85°. a, In the region of low obliquity, there exists a large island corresponding to resonances with the secular frequency $s_6=-26.3302$ arcsec yr^{-1} of the node of Jupiter and Saturn. This region is also visible in (b).

BOX 2 Analysis of resonances

The solution of the orbital motion of the Earth being chaotic[4,11-14], a quasiperiodic approximation of $A(t)+iB(t)$ in equation (4), is not well suited for obtaining accurate solutions over a few million years, but will be useful for a qualitative understanding of the behaviour of the solution. In the Fourier spectrum of $A(t)+iB(t)$ (Fig. 1) we can observe peaks which are identified as the main planetary secular frequencies in inclination. Around each of these, several secondary peaks appear, which largely reflect the non-regular behaviour of the solution. A frequency analysis[4-6] of $A(t)+iB(t)$ can be done in order to find a quasiperiodic approximation of this function over a few million years of the form

$$A(t)+iB(t)\approx\sum_{k=1}^{N}\alpha_k\, e^{i(\nu_k t+\phi_k)}$$

With this approximation, the hamiltonian now reads

$$H=\tfrac{1}{2}\alpha X^2+\sqrt{1-X^2}\sum_{k=1}^{N}\alpha_k\sin(\nu_k t+\psi+\phi_k) \qquad (5)$$

which is the hamiltonian of an oscillator of frequency αX, perturbed by a quasiperiodic external oscillation with several frequencies ν_k. Resonance will occur when $\dot\psi\approx\alpha X=\alpha\cos\varepsilon$ is equal to the opposite $(-\nu_k)$ of one of the frequencies ν_k.

The value for the mean precession speed of the Earth over 18 Myr is $p_f=50.4712''\,\mathrm{yr}^{-1}$. This value is very close to the opposite of a small term due to the opposite of a small term due to the perturbations of Jupiter and Saturn $s_6-g_6+g_5=-50.3021''\,\mathrm{yr}^{-1}$. The passage through resonance could occur during an ice age, and could lead to an increase of 0.5° in the obliquity variations[7]. Nevertheless, the Earth is far from the main planetary resonances, the closest being $s_6=-26.3302''\,\mathrm{yr}^{-1}$. With the current value of α, we estimate that this resonance is reached for an obliquity of about 60°.

Thus, we find that even if the initial obliquity of the Earth was very small, resonances or chaotic behaviour could have raised it to 50° in a few million years, with mean value 20°–30°. If the Moon was then captured, the precession frequency would have increased suddenly, and the motion would have become regular. The value of the obliquity would be frozen to its current value and thereafter suffer only small oscillations, until tidal dissipation[3] ultimately drives the Earth into the large chaotic zone.

It can thus be claimed that the Moon is a climate regulator for the Earth. If it were not present, or if it were much smaller, for many values of the Earth primordial spin rate, the obliquity of the Earth would be chaotic with very large variations, reaching more than 50° in a few million years and even, in the long term, more than 85°. This would probably have drastically changed the climate on the Earth. □

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