Section 6.2 - Properties of Logarithms

Remember the properties of exponents?

	Properties of exponents	Let $f(x) = b^x$. Write the property using function notation.	Interchange the role of outputs and inputs to find similar rules for $f^{-1}(x)$.	Let $f^{-1}(x) = \log_b(x)$ to get the Properties of Logarithms
Product Rule	$b^u b^w = b^{u+w}$	$f(u)f(w) = f(u+w)$	$f^{-1}(uw) = f^{-1}(u) + f^{-1}(w)$	$\log_b(uw) = \log_b u + \log_b w$
Quotient Rule	$\frac{b^u}{b^w} = b^{u-w}$	$\frac{f(u)}{f(w)} = f(u-w)$	$f^{-1}\left(\frac{u}{w}\right) = f^{-1}(u) - f^{-1}(w)$	$\log_b\left(\frac{u}{w}\right) = \log_b u - \log_b w$
Power Rule	$(b^u)^w = b^{uw}$	$(f(u))^w = f(uw)$	$w f^{-1}(u) = f^{-1}(u^w)$	$w\log_b u = \log_b(u^w)$

Example 1: Expand the following using the properties of logarithms and simplify.

a. $\log_2\left(\frac{8}{x}\right)$

$\log_2 8 - \log_2 x$

$-\log_2 x + 3$

b. $\log_{0.1}(10x^2)$

$\log_{0.1} 10 + \log_{0.1} x^2$

$2\log_{0.1} x - 1$

c. $\ln\left(\left(\frac{3}{ex}\right)^2\right)$

$2\ln\left(\frac{3}{ex}\right) =$

$2\ln 3 - 2\ln(ex)$

$2\ln 3 - 2\ln e - 2\ln x$

$-2\ln x + 2\ln 3 - 2$

d. $\log\sqrt[3]{\frac{100x^2}{yz^5}}$

$\frac{1}{3}\log\left(\frac{100x^2}{yz^5}\right)$

e. $\log_{117}(x^2 - 4)$

$\log_{117}(x+2) + \log_{117}(x-2)$

Example 2: Use the properties of logarithms to write the following as a single logarithm.

a. $\log_3(x-1) - \log_3(x+1)$

$\log_3\left(\frac{x-1}{x+1}\right)$

b. $\log(x) - 2\log(y) - \log(z)$

$\log x - \log y^2 - \log z$

$\log\left(\frac{x}{y^2 z}\right)$

c. $4\log_2(x) + 3$

$\log_2 x^4 + \log_2 8$

$\log_2(8x^4)$

d. $-\ln(x) - \frac{1}{2}$

$\ln\left(\frac{1}{x}\right) - \ln e^{1/2}$

$\ln\left(\frac{1}{\sqrt{e}\,x}\right)$

Note: The unwritten property of logarithms is that if it isn't written in a textbook, it probably isn't true.	
The two logarithm buttons commonly found on calculators are the `LOG' and `LN' buttons which correspond to the common and natural logs. How might we find an approximation for logs of bases other than 10 and *e*, for example $log_2(7)$?	
Change of base formula:	Examples:
$log_a(x) = \frac{log_b(x)}{log_b(a)}$ $\frac{\log X}{\log A}$ $\frac{\ln X}{\ln A}$	a. Change $log_2(7)$ to base 10. $\frac{\log 7}{\log 2}$ b. Change $f(x) = log_4(x)$ to a function with base e $\frac{\ln x}{\ln 4}$
$a^x = b^{x\,log_b(a)}$ OR $a^x = b^{log_b(a^x)}$	a. Change 6^3 to base 10. $10^{\log 6^3} = 10^{3 \log 6}$ b. Change the number "5" to an exponent with base "e" $e^{\ln 5}$ c. Change $f(x) = 5^x$ to a function with base e. $e^{\ln 5^x} = e^{x \ln 5}$
Note: The change of base formulas really tells us is that all exponential and logarithmic functions are just scalings of one another. Not only does this explain why their graphs have similar shapes, but it also tells us that we could do all of math with a single base.	