17.3 Using a Pythagorean Identity

Essential Question: How can you use a given value of one of the trigonometric functions to calculate the values of the other functions?

Resource Locker

Explore Proving a Pythagorean Identity

In the previous lesson, you learned that the coordinates of any point (x, y) that lies on the unit circle where the terminal ray of an angle θ intersects the circle are $x = \cos\theta$ and $y = \sin\theta$, and that $\tan\theta = \frac{y}{x}$. Combining these facts gives the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$, which is true for all values of θ where $\cos\theta \neq 0$. In the following Explore, you will derive another identity based on the Pythagorean theorem, which is why the identity is known as a *Pythagorean identity*.

(A) The terminal side of an angle θ intersects the unit circle at the point (a, b) as shown. Write a and b in terms of trigonometric functions involving θ.

$a = \cos\theta$

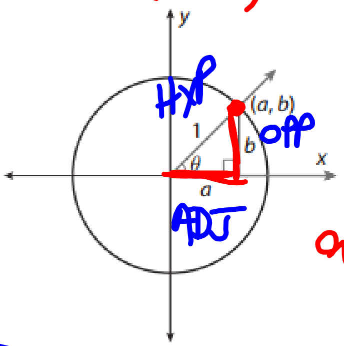
$b = \sin\theta$

(x, y)

$(\cos\theta, \sin\theta)$

$\cos\theta = \frac{a}{1}$

$\sin\theta = \frac{b}{1}$



$a^2 + b^2 = 1^2$

(B) Apply the Pythagorean theorem to the right triangle in the diagram. Note that when a trigonometric function is squared, the exponent is typically written immediately after the name of the function. For instance, $(\sin\theta)^2 = \sin^2\theta$.

$(\cos\theta)^2 \neq \cos\theta^2$

$(\cos\theta)^2 = \cos^2\theta$ ✓

Write the Pythagorean Theorem. $a^2 + b^2 = c^2$

Substitute for a, b, and c. $(\cos\theta)^2 + (\sin\theta)^2 = (1)^2$

Square each expression. $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta + \sin^2\theta = 1$

Reflect

1. The identity is typically written with the sine function first. Write the identity this way, and explain why it is equivalent to the one in Step B.

2. Confirm the Pythagorean identity for $\theta = \frac{\pi}{3}$.

$\cos^2\left(\frac{\pi}{3}\right) + \sin^2\left(\frac{\pi}{3}\right) = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

$\frac{1}{4} + \frac{3}{4} = 1$ ✓

3. Confirm the Pythagorean identity for $\theta = \frac{3\pi}{4}$.

$\cos^2\left(\frac{3\pi}{4}\right) + \sin^2\left(\frac{3\pi}{4}\right) = 1$

$\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$

$\frac{1}{2} + \frac{1}{2} = 1$

Explain 1 Finding the Value of the Other Trigonometric Functions Given the Value of $\sin\theta$ or $\cos\theta$

You can rewrite the identity $\sin^2\theta + \cos^2\theta = 1$ to express one trigonometric function in terms of the other. As shown, each alternate version of the identity involves both positive and negative square roots. You can determine which sign to use based on knowing the quadrant in which the terminal side of θ lies.

Solve for $\sin\theta$	Solve for $\cos\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$	$\cos^2\theta = 1 - \sin^2\theta$
$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$	$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

Example 1 **Find the approximate value of each trigonometric function.**

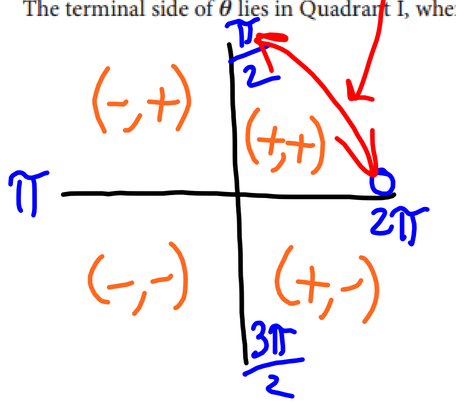
(A) Given that $\sin\theta = 0.766$ where $0 < \theta < \frac{\pi}{2}$, find $\cos\theta$.

Use the identity to solve for $\cos\theta$. $\quad \cos\theta = \pm\sqrt{1 - \sin^2\theta}$

Substitute for $\sin\theta$. $\quad = \pm\sqrt{1 - (0.766)^2}$

Use a calculator, then round. $\quad \approx \pm 0.643$

The terminal side of θ lies in Quadrant I, where $\cos\theta > 0$. So, $\cos\theta \approx 0.643$.



(x, y)

$(\cos\theta, \sin\theta)$

(B) Given that $\cos\theta = -0.906$ where $\pi < \theta < \frac{3\pi}{2}$, find $\sin\theta$.

Use the identity to solve for $\sin\theta$. $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$

Substitute for $\cos\theta$. $= \pm\boxed{\sqrt{1 - (-.906)^2}}$

Use a calculator, then round. $\approx \pm .423$

The terminal side of θ lies in Quadrant 2, where $\sin\theta$ ☐ 0. So, $\sin\theta = -.423$

Reflect

4. Suppose that $\frac{\pi}{2} < \theta < \pi$ instead of $0 < \theta < \frac{\pi}{2}$ in part A of this Example. How does this affect the value of $\sin\theta$?

5. Suppose that $\frac{3\pi}{2} < \theta < 2\pi$ instead of $\pi < \theta < \frac{3\pi}{2}$ in part B of this Example. How does this affect the value of $\sin\theta$?

6. Explain how you would use the results of part A of this Example to determine the approximate value for $\tan\theta$. Then find it.

Your Turn

7. Given that $\sin\theta = -0.644$ where $\pi < \theta < \frac{3\pi}{2}$, find $\cos\theta$.

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$\sqrt{1-(-.644)^2}$

.7650254

8. Given that $\cos\theta = -0.994$ where $\frac{\pi}{2} < \theta < \pi$, find $\sin\theta$. Then find tar

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$\sqrt{1-(-.994)^2}$

.1093800713

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