# MTHM 171 College Algebra - Final Exam Study Guide

Please review exams, homework problems, class notes, and text. Pay special attention to the following topics. Note: As on your midterms you MUST show appropriate work. I will not give credit for solutions derived with incorrect or incomplete work. Also, be careful about your notation!

1. Use the distance and midpoint formula.
2. Given the graph of a function
   a. State the domain/range
   b. Find the interval(s) on which the function is increasing/decreasing
   c. Find the local maximum(s)/minimum(s), absolute maximum(s)//minimums
   d. List all of the asymptote (use correct notation)
   e. List all of the intercepts (use correct notation)
   f. Determine the behavior of f(x) at an asymptote. (as $x \to 2^+$ , $f(x) \to ?$)
   g. Determine end behavior (as $x \to \infty$, $f(x) \to ?$)
3. Use function notation. What does *f*(7) mean?
4. Simplify the difference quotient.
5. Determine whether an equation/graph defines *y* as a function of *x*.
6. Find the implied domains of functions. Give answers in interval notation.
7. Determine whether a function is even or odd.
8. Sketch the graphs of functions.
   a. Piecewise
   b. Linear
   c. Absolute Value
   d. Quadratic (find the vertex)
   e. Polynomial
   f. Rational (domain, vertical asymptotes, holes, horizontal asymptotes, slant asymptotes, x- and y-intercepts, sign diagrams)
   g. Algebraic (involving roots)
   h. Logarithmic
   i. Exponential
   j. Inverse Functions
9. Predict the affect that function transformations have on a graph.
10. Find all the zeros (real or complex) of a polynomial and their multiplicities. Factor the polynomial.
11. Function composition/decomposition.
12. Find the inverse function.
13. Solve equations.
    a. Absolute value
    b. Quadratic
    c. Rational
    d. Algebraic
    e. Exponential
    f. Logarithmic
14. Solve inequalities using a sign chart. Give answers in interval notation.
    a. Absolute value
    b. Quadratic
    c. Polynomial
    d. Rational
    e. Algebraic
    f. Exponential
    g. Logarithmic

15. Solve application problems.
    a. Applications of Linear Functions (constant rate of change)
    b. Find the Profit function given the demand function and the cost function. Find the maximum profit.
    c. Radioactive decay/Population growth
    d. Compound Interest
    e. Systems of Equations.
16. Use properties of logarithms to rewrite expressions involving logarithms.
17. Find the equation of a circle.
18. Solve systems of linear equations with two or more variables.
19. Perform matrix arithmetic.
20. Find the inverse of a matrix.
21. Solve a system of equations using the inverse of a matrix.
22. Find the partial fraction decomposition of a rational expression.

**Some Formulas to remember:**

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) \qquad m=\frac{y_1-y_0}{x_1-x_0}$$

$$\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right) \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$(f^{-1}\circ f)(x)=x \qquad b^{\log_b x}=x \qquad \log_b b^x=x \qquad b^a=c\leftrightarrow\log_b c=a$$

$$\log_b(xy)=\log_b x+\log_b y \qquad \log_b(x/y)=\log_b x-\log_b y \qquad \log_b x^n=n\log_b x$$

$$y(t)=y_0e^{kt} \qquad AA^{-1}=A^{-1}A=I_n$$

## Sample Questions

1. Find the equation of a circle whose diameter has end points (-1, 2) and (3, 8).

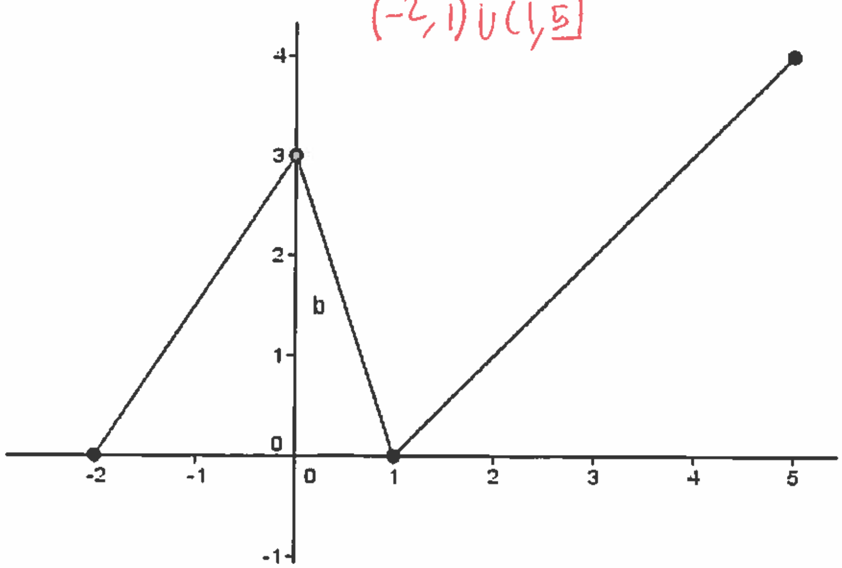
$h = \frac{-1+3}{2} = 1$ $\quad R = \sqrt{(-1-3)^2+(2-8)^2}$

$K = \frac{2+8}{2} = 5$ $\quad R = \sqrt{52}$

$(x-1)^2+(y-5)^2 = 52$

2. Sketch the graph of a function *f* that has the desired characteristics.
   - $f(2) = 6$
   - As $x \to \infty, f(x) \to -1$
   - The point (1, 3) is a hole.
   - The point (-2, 7) is a local maximum on the graph of $y = f(x)$.
   - As $x \to 4^+$ , $f(x) \to -\infty$

3. Given the graph of *f* below.
   - State the domain/range D: [-2,5] R: [0,4]
   - Find the interval(s) on which the function is increasing/decreasing
     Inc [-2,0] ∪ [1,5] Dec [0,1]
   - Find the local maximum(s)/minimum(s), absolute maximum(s)/minimums
     Loc max: (0,3) & (5,4) Loc min (-2,0) & (1,0) Abs max (5,4) Abs min (-2,0)(1,0)
   - List all of the intercepts (use correct notation)
     (-2,0) (1,0) (0,3)
   - Find $f(5)$.
     $f(5) = 4$
   - Approximate $f(f(5))$. $f(f(5)) = f(4) = 3$
   - Solve $f(x) = 0$.
     $x = -2$ or $x = 1$
   - Solve $f(x) > 0$.
     $(-2, 1) \cup (1, 5]$



4. Simplify the following difference quotient $\frac{f(x+h)-f(x)}{h}$

- $f(x) = 7$

$\frac{7-7}{h} = \frac{0}{h} = 0$

- $f(x) = 2x^2 + 9$

$f(x+h) = 2(x+h)^2 + 9$
$= 2(x+h)(x+h) + 9$
$= 2(x^2+2hx+h^2)+9 = 2x^2+4hx+2h^2+9$

$\frac{2x^2+4hx+2h^2+9-(2x^2+9)}{h} = \frac{4hx+2h^2}{h} = 4x+2$

- $f(x) = \frac{x}{x+2}$

$f(x+h) = \frac{x+h}{x+h+2}$

$\frac{\frac{(x+h)(x+2)}{(x+h+2)(x+2)} - \frac{x(x+h+2)}{(x+2)(x+h+2)}}{h} = \frac{\frac{x^2+2x+hx+2h-x^2-hx-2x}{(x+2)(x+h+2)}}{h} = \frac{\frac{2h}{(x+2)(x+h+2)}\left(\frac{1}{h}\right)}{h\left(\frac{1}{h}\right)} = \frac{2}{(x+2)(x+h+}$

- $f(x) = \sqrt{x-3}$

$f(x+h) = \sqrt{x+h-3}$

$\frac{\sqrt{x+h-3} - \sqrt{x-3}}{h}$

5. Determine whether the equation defines $y$ as a function of $x$.

- $x^2 + y^2 = 1 \rightarrow y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

NO, PLUS/MINUS MEANS IT TAKES. TWO FUNCTIONS TO DEFINE

- $11 = y^3 - x$

$11 + x = y^3$ $\sqrt[3]{11+x} = y$ Yes

6. Find the implied domains of functions. Give answers in interval notation.

- $f(x) = 2x^2 + 9$

$(-\infty, \infty)$

- $f(x) = \frac{x}{x+2}$

$x+2=0$
$x=-2$

$(-\infty,-2)(-2,\infty)$

- $f(x) = \sqrt{3-x}$

$3-x=0$
$3=x$

$f(x)$ +++0- - -

$x$ 3

$(-\infty, 3]$

- $f(x) = log(x+2)$

$x+2=0$
$x=-2$

$f(x)$ - - 0 +++

$x$ -2

$(-2, \infty)$

- $f(x) = \frac{log(x+2))}{\sqrt{3-x}}$

$(-2, 3)$

7. Determine whether the following functions are even, odd, or neither. Explain.

- $f(x) = x^2 + 11$

$f(-x) = (-x)^2 + 11$
$f(-x) = x^2 + 11$

EVEN $f(x) = f(-x)$

- $f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$
$f(-x) = -x^3 + x$
$f(-x) \neq f(x)$

$-f(x) = -(x^3 - x)$
$-f(x) = -x^3 + x$

ODD $f(-x) = -f(x)$

- $f(x) = x^2 + 2x - 1$

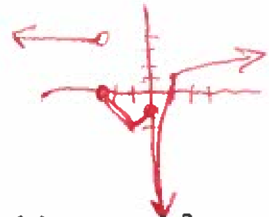
$f(-x) = (-x)^2 + 2(-x) - 1$
$f(-x) = x^2 - 2x - 1$

$-f(x) = -(x^2+2x-1)$
$-f(x) = -x^2 - 2x + 1$
$f(-x) \neq -f(x)$

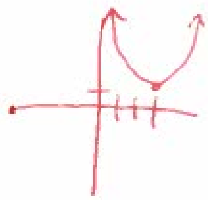
Neither

8. Sketch the graphs of the following functions.

- $f(x) = \begin{cases} 4 & if \quad x < -3 \\ |x+1| - 2 & if \quad -3 \le x \le 0 \\ log_3(x) + 1 & if \quad x > 0 \end{cases}$



- $g(x) = (x-3)^2 + 1$



- $h(x) = -(x-1)^2(x+2)(x-7)^4$

- $j(x) = \frac{-(x-2)(x+7)}{(x-2)(x+3)}$

9. Suppose (-3,4) is on the graph of $y = f(x)$. Find a point on the graph of the given transformed function.

- $f(x-3)$ (0,4)
- $f(2x)$ $(-\frac{3}{2},4)$
- $f(-x)$ (3,4)
- $f(x)+4$ (-3,8)
- $5f(x)$ (-3,20)
- $-f(x)$ (-3,-4)
- $-5f(2x+3)-9$ (-3,-29)
- $f\left(\frac{2-x}{4}\right)$ (14,4)

10. Let $f(x) = 2x^4 + 4x^2 + 16x + 10$. Find all the zeros of *f*. Then factor *f* completely over the complex numbers and the real numbers.

$x^4 + 2x^2 + 8x + 5$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & 2 & 8 & 5 \\ & & -1 & 1 & -3 & -5 \\ \hline -1 & 1 & -1 & 3 & 5 & 0 \\ & & -1 & 2 & -5 & \\ \hline & 1 & -2 & 5 & 0 & \end{array}$$

$(x+1)^2(x^2-2x+5) \leftarrow$ FACTOR OVER REAL

11. Let $f(x) = 2x^2 + 9, g(x) = \frac{x}{x+2}$, and $h(x) = \sqrt{x-3}$. Find and simplify $g\left(f\left(h(x)\right)\right)$.

$f(h(x)) = 2(\sqrt{x-3})^2 + 9$
$= 2(x-3) + 9$
$= 2x + 3$

$g(f(h(x))) = \frac{2x+3}{2x+3+2}$

$g(f(h(x))) = \frac{2x+3}{2x+5}$

12. Find the inverse of the following functions.

- $f(x) = 2x^2 + 9, x > 0$

$y = 2x^2 + 9$
$y - 9 = 2x^2$
$\frac{y-9}{2} = x^2$
$\pm\sqrt{\frac{x-9}{2}} = x$

$f^{-1}(x) = \sqrt{\frac{x-9}{2}}$

- $f(x) = \frac{x}{x+2}$

$y = \frac{x}{x+2}$
$xy + 2y = x$
$xy - x = 2y$
$x(y-1) = 2y$

$x = \frac{2y}{y-1}$

$f^{-1}(x) = \frac{2x}{x-1}$

- $f(x) = x^2 + 6x - 1, x < -3$

$y = x^2 + 6x - 1$
$y + 9 = x^2 + 6x + 9 - 1$
$y + 9 = (x+3)^2 - 1$
$y + 10 = (x+3)^2$

$\pm\sqrt{y+10} = x + 3$
$-3 \pm \sqrt{y+10} = x$

$f^{-1}(x) = -3 - \sqrt{x+10}$

- $f(x) = -log_4(x-7) + 3$

$y = -\log_4(x-7) + 3$
$y - 3 = -\log_4(x-7)$
$-y + 3 = \log_4(x-7)$
$4^{-y+3} = x - 7$
$4^{-y+3} + 7 = x$

$f^{-1}(x) = 4^{-y+3} + 7$

13. Solve the following equations.

- $|x-7|-1=4$

  $|x-7|=5$  $x-7=5$  $x=12$  $x-7=-5$  $x=2$

- $2x^2+3x=-10$

  $2x^2+3x+10=0$  $\frac{-3\pm\sqrt{9-4(2)(10)}}{2(2)}=\frac{-3\pm\sqrt{-71}}{4}=\frac{-3\pm i\sqrt{71}}{4}$

- $2x^4+4x^2=-16x-10$

  $2x^4+4x^2+16x+10=0$

  $x^4+2x^2+8x+5=0$  SEE #10 $\rightarrow (x+1)^2(x^2-2x+5)=0$

  $(x+1)^2=0$  $x+1=0$  $x=-1$

  $\frac{2\pm\sqrt{4-4(1)(5)}}{2(1)}=\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm4i}{2}=$  $x=1\pm2i$

- $\left(\frac{1}{x+2}+\frac{2}{x-7}=\frac{-1}{x+2}\right)(x+2)(x-7)$

  $x-7+2x+4=-x+7$

  $4x=10$

  $x=5/2$

- $(\sqrt{x+7})^2=(\sqrt{x}+1)^2$

  $x+7=x+2\sqrt{x}+1$

  $6=2\sqrt{x}$

  $3=\sqrt{x}$

  $9=x$

- $2^{x+1}=9\cdot3^{2x-1}$

  $2^{x+1}=3^2\cdot3^{2x-1}$

  $2^{x+1}=3^{2x+1}$

  $\ln 2^{x+1}=\ln 3^{2x+1}$

  $(x+1)\ln2=(2x+1)\ln3$

  $x\ln2+\ln2=2x\ln3+\ln3$

  $x\ln2-2x\ln3=\ln3-\ln2$

  $x(\ln2-2\ln3)=\ln3-\ln2$

  $x=\frac{\ln3-\ln2}{\ln2-2\ln3}$

  $x=\frac{\ln(3/2)}{\ln(2/9)}$

- $\log(x+1)=\log(2x)+1$

  $\log\left(\frac{x+1}{2x}\right)=1$

  $\frac{x+1}{2x}=10$

  $x+1=20x$

  $1=19x$

  $\frac{1}{19}=x$

- $\log(x+1)+\log(4)=\log(x-3)$

  $\log(4x+4)=\log(x-3)$

  $4x+4=x-3$

  $3x=-7$

  $x=\frac{-7}{3}$ Not on domain of $\log(x+1)$

  No Solution

- $log_9(x)=log_3(2x+9)$

  $\log_9 x=\log_9(4x^2+36x+81)$

  $x=4x^2+36x+81$

  $0=4x^2+35x+81$

  $\frac{-35\pm\sqrt{35^2-4\cdot4\cdot81}}{2\cdot8}=\frac{-35\pm\sqrt{-71}}{16}=\frac{-35\pm i\sqrt{71}}{16}$

14. Solve the following inequalities.

- $|x-7|-1>4$

  $|x-7|>5$ ← Great "OR"

  $x-7>5$ OR $x-7<-5$

  $x>12$ OR $x<2$

- $|x-7|-1\le 4$

  $|x-7|\le 5$ ← Less "THAND"

  $-5\le x-7\le 5$

  $2\le x\le 12$

- $2x^2+3x<-10$

  $2x^2+3x+10<0$ ← Always Positive

  $\frac{-3\pm\sqrt{9-4(2)(10)}}{2(2)}=\frac{-3\pm\sqrt{-71}}{4}$ ← Imag

  No Solution

- $2x^4+4x^2>-16x-10$

  $2x^4+4x^2+16x+10>0$

  From #10 → $(x+1)^2(x^2-2x+5)>0$

  From #13 → $x=-1$ $\quad x=1\pm 2i$

  $f(x)$: + + + + 0 + + + + +; $x$: $-1$

  $(-\infty,0)(0,\infty)$

- $\frac{1}{x+2}+\frac{2}{x-7}<\frac{-1}{x+2}$

  $\frac{2}{x+2}+\frac{2}{x-7}<0$

  From 13 → $x=5/2$

  $f(x)$: - - - ¿ + + 0 - - - ¿ + + +; $x$: $-2$, $5/2$, $7$

  $(-\infty,-2)\cup(5/2,7)$

- $\sqrt{x+7}>\sqrt{x}+1$

  From #13 → $x=9$

  $\sqrt{x+7}-\sqrt{x}-1>0$

  $f(x)$: + + + + 0 - - - - -; $x$: $0$, $9$

  $[0,9)$

- $2^{x+1}>9\cdot 3^{2x-1}$

  $2^{x+1}-9\cdot 3^{2x-1}>0$

  From #13 → $x=\frac{\ln(3/2)}{\ln 4/9}\approx -.2696$

  $f(x)$: + + + + + 0 - - - - -; $x$: $\frac{\ln(3/2)}{\ln(4/9)}$

  $\left(-\infty,\frac{\ln(3/2)}{\ln(4/9)}\right)$

- $\log(x+1)<\log(2x)+1$

  $\log(x+1)-\log(2x)-1<0$

  From #13 → $x=1/19$

  $f(x)$: + + + 0 - - - - -; $x$: $0$, $1/19$

  $\left(\frac{1}{19},\infty\right)$

- $\log(x+1)+\log(4)>\log(x-3)$

  $\log(x+1)+\log 4-\log(x-3)>0$

  From #13 → No Solution

  $f(x)$: DNE ¿ + + + + + + + + +; $x$: $3$

  $(3,\infty)$

15. Solve the following application problems.

- A math tutor charged \$22 for two hours of tutoring and \$49 for five hours of tutoring. Find a linear function which models the cost of tutoring *C* as a function of time *t* in hours. Put your answer in **point-slope form** and **slope-intercept form**.

- A group of students decided to raise money to buy their teacher a thank-you-gift. To do this, they made origami calculators. It has been determined that the cost in dollars of making $x$ origami calculators is $C(x) = .01x + 4$. And that the price-demand function for the origami calculators is $p(x) = 4 - 0.01x$.
  i. Find and simplify an expression for the revenue *R* as a function of *x*.

  ii. Find and simplify an expression for the profit *P* as a function of *x*.

  iii. How many origami calculators should they make in order to maximize their profit?

  iv. What is the maximum profit?

- Mathium 314, used to power very expensive graphing calculators, has an initial mass of approximately 22/7 milligrams. It has a half-life of approximately 2.7 days.
  i. Find a function which gives the amount of Uranium 235 which remains after time *t*.

  ii. Determine how long it takes for 10% of the material to decay.

- \$2000 is invested in an account which offers 1.125% compounded monthly.
  i. Determine how much is in the account in 5 years.

  ii. How long will it take for the initial investment to double?

- A student needs to study for three comprehensive final exams: Mathematics, Chemistry, and Psychology. The student had a total of 100 ounces of energy drink that she could use to help her stay alert as she studied. To help her study, the student felt like she needed 2 ounces of energy drink for every hour she spent studying chemistry and 3 ounces of energy drink for every hour she spent studying psychology. She did not need any energy drink when she studied mathematics because studying mathematics was fun and energizing on its own. The student realized mathematics was fun, but challenging, so she wanted the amount of time she studied mathematics to equal the amount of time she studied for her other two exams combined. She has a total of 50 hours to study. How much time should she spend studying for each exam?

16. Solve the following systems of equations.

- $\begin{cases} 3x + 3y + 6z = 4 \\ 6x + 8y + 2z = 5 \end{cases}$

$$\begin{bmatrix} 3 & 3 & 6 & 4 \\ 6 & 8 & 2 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 2 & 4/3 \\ 0 & 2 & -10 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 7 & 17/6 \\ 0 & 1 & -5 & -3/2 \end{bmatrix}$$

$$\left(-7t + 17/6,\ 5t - 3/2,\ t\right)$$

- $\begin{cases} 5x + 2y + 19z = 17 \\ 2x + y + 8z = 8 \\ 3x + y + 11z = 14 \end{cases}$

$$\begin{bmatrix} 5 & 2 & 19 & 17 \\ 2 & 1 & 8 & 8 \\ 3 & 1 & 11 & 14 \end{bmatrix} \to \begin{bmatrix} 1 & 2/5 & 19/5 & 17/5 \\ 0 & 1/5 & 2/5 & 6/5 \\ 0 & -1/5 & -2/5 & 19/5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

NO Solution!

- $\begin{cases} -2x - 4y + z = 13 \\ 7x + 13y - 2z = -35 \\ -x - 2y + z = 9 \end{cases}$

$$\begin{bmatrix} -2 & -4 & 1 & 13 \\ 7 & 13 & -2 & -35 \\ -1 & -2 & 1 & 9 \end{bmatrix} \to \begin{bmatrix} -1 & -2 & 1 & 9 \\ 7 & 13 & -2 & -35 \\ -2 & -4 & 1 & 13 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 & -9 \\ 0 & -1 & 5 & 28 \\ 0 & 0 & -1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 9 & 47 \\ 0 & 1 & -5 & -28 \\ 0 & 0 & 1 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \to (2, -3, 5)$$

17. Perform the following matrix operations if possible.

- $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 5 \\ 2 & 3 \end{bmatrix}$ $\quad \begin{bmatrix} 8 & 17 \\ 10 & 25 \\ 12 & 33 \end{bmatrix}$

- $\begin{bmatrix} 0 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ NOT Possible

- $\begin{bmatrix} 0 & 5 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ NOT Possible

- $2\begin{bmatrix} 0 & 5 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 10 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 2 & 12 \end{bmatrix}$

18. Find the inverse of matrix $A$ below. Use it to solve the following system of equations.

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 4 & 8 & 4 \\ 1 & 0 & -1 \end{bmatrix} \qquad \begin{cases} 2y + 3z = 1 \\ 4x + 8y + 4z = 0 \\ x - z = 2 \end{cases}$$

$$\left[\begin{array}{ccc|ccc} 0 & 2 & 3 & 1 & 0 & 0 \\ 4 & 8 & 4 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

switch $R_1$ & $R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 4 & 8 & 4 & 0 & 1 & 0 \\ 0 & 2 & 3 & 1 & 0 & 0 \end{array}\right]$$

$-4R_1$ & $R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 8 & 8 & 0 & 1 & -4 \\ 0 & 2 & 3 & 1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1/8 & -1/2 \\ 0 & 0 & 1 & 1 & -1/4 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/4 & 2 \\ 0 & 1 & 0 & -1 & 3/8 & -3/2 \\ 0 & 0 & 1 & 1 & -1/4 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -1/4 & 2 \\ -1 & 3/8 & -3/2 \\ 1 & -1/4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1/4 & 2 \\ -1 & 3/8 & -3/2 \\ 1 & -1/4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$(5, -4, 3)$

19. Find only the form needed to begin the process of partial fraction decomposition.

$$\frac{2}{(x-3)^4(x+1)(x^2+13)^2}$$

$$\frac{A}{X-3} + \frac{B}{(X-3)^2} + \frac{C}{(X-3)^3} + \frac{D}{(X-3)^4} + \frac{E}{X+1} + \frac{FX+G}{X^2+13} + \frac{HX+I}{(X^2+13)^2}$$

20. Find the partial fraction decomposition of the following rational expression.

$$\frac{16-3x}{(x-2)^2(x+3)}$$

$$\frac{16-3X}{(X-2)^2(X+3)} = \frac{A\,(X-2)(X+3)}{(X-2)(X-2)(X+3)} + \frac{B\,(X+3)}{(X-2)^2(X+3)} + \frac{C\,(X-2)^2}{(X+3)(X-2)^2}$$