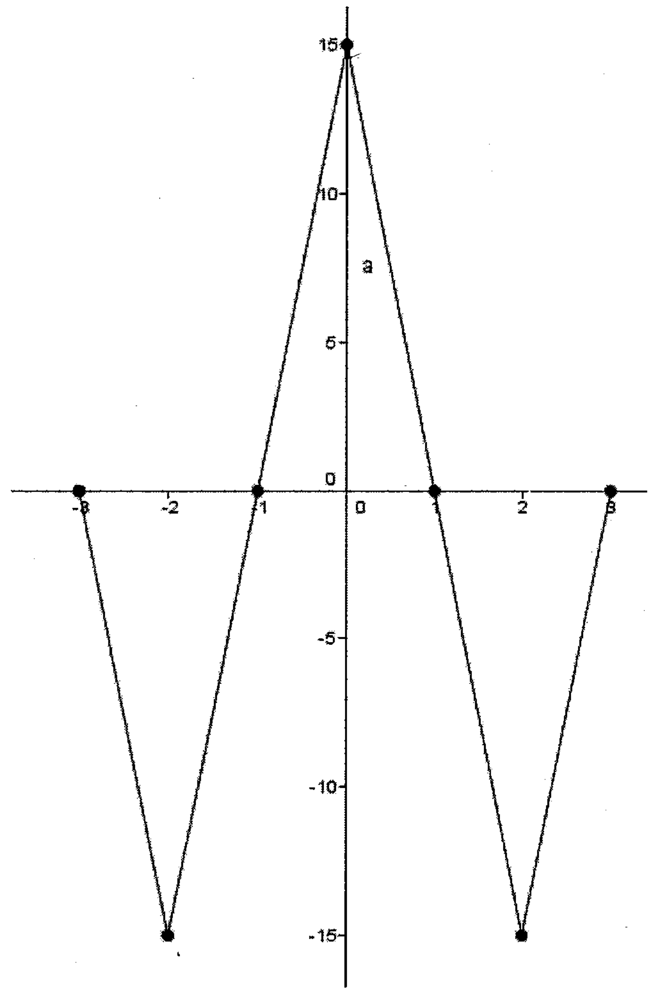
MTHM 171 **Practice Exam 1** Name: Solutions

Use the methods we discussed in class to do the following problems. With the exception of problems 2, 3, and 9a your answers MUST be accompanied by work or an explanation, which will be graded on its clarity, accuracy, and completeness. **Your work should be detailed enough to convince me that you can do these problems without a graphing calculator**. Good luck!

1. Notation/Clarity: 4 3 2 1 0 (For instructor use only. Please leave this blank.)

2. Let f be the function whose graph is given below. Answer the following questions using the correct notation.



x-values → (a) List the interval(s) where f is increasing?

$[-2,0]$, $[2,3]$

(b) List the local maximum(s) if any exist. If none exist, write DNE.

(x,y) → $(0,15)$

(c) Find the absolute maximum, if it exists. If none exist, write DNE.

y → $y=15$

(d) State the domain of f using interval notation.

x-values → $[-3,3]$

(e) State the range of f using interval notation.

y → $[-15,15]$

(f) List the x-intercept(s), if any exist.

$(x,0)$ → $(-3,0), (-1,0), (1,0), (3,0)$

(g) List the zeros, if any exist.

x-values $x=-3,-1,1,3$

$f(0)$ is a y-value (h) Determine $f(0)$.

$f(0)=15$

Find x when $y=-15$ (i) Solve $f(x) = -15$

$x=-2,2$

Find x when $y \geq 0$ (j) Solve $f(x) \geq 0$

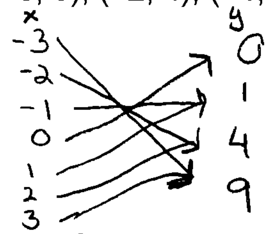
$\{-3,3\}$, $[-1,1]$

(k) Does f appear to be even, odd, or neither?

Even $f(x)=f(-x)$ for all x.

3. Determine whether or not the relation {(−3, 9), (−2, 4), (−1, 1), (0, 0), (1, 1), (2, 4), (3, 9)} represents y as a function of x.

y <u>IS</u> a function of x.



Notice, for all x, there is exactly one y. Ex. -3 only corresponds to 9.

4. Determine whether or not the equation $x = y^2 + 4$ represents y as a function of x

$$x-4=y^2$$

$$y=\pm\sqrt{x-4}$$

y is NOT a function of x. Ex. $\cancel{X}=5$ corresponds to $y=\pm1$

5. f is a function that takes a real number x and performs the following three steps in the order given: (1) add 3; (2) take the square root; (3) multiply by 2. Find an expression for $f(x)$.

$$f(x)=2\sqrt{x+3}$$

6. Let $f(x) = x^2 + x$, $g(x) = \frac{x}{2x+3}$, and $h(x) = 6x$, $k(x) = 7$.

(a) Find and simplify the following:

i. $f(3) =$ $3^2+3=\boxed{12}$

ii. $f(x) - 3 =$ $(x^2+x)-3=\boxed{x^2+x-3}$

iii. $f(x - 3) =$ $(x-3)^2+(x-3)=$ ↓ FOIL $x^2-6x+9+x-3=\boxed{x^2-5x+6}$

iv. $f(-x) =$ $(-x)^2-x=\boxed{x^2-x}$

v. $k(2x) = \boxed{7}$

(b) Compute $(gf)(-3)$.

$$(gf)(-3)=g(-3)f(-3)$$

$$=\left(\frac{-3}{2(-3)+3}\right)\left((-3)^2+-3\right)=\left(\frac{-3}{-3}\right)(6)=\boxed{6}$$

(c) Find and simplify a formula for $\left(\frac{g}{h}\right)(x)$.

$$\left(\frac{g}{h}\right)(x)=\frac{\frac{x}{2x+3}}{6x}=\frac{\frac{x}{\cancel{2x+3}}\cdot(2x+3)}{\frac{6x}{1}\cdot(2x+3)}=\frac{\cancel{x}}{6\cancel{x}(2x+3)}=\frac{1}{12x+18}$$

(d) Find the implied domain of $\left(\frac{g}{h}\right)(x)$.

$$\left(\frac{g}{h}\right)(x)=\frac{g(x)}{h(x)}=\frac{\frac{x}{2x+3}}{6x}$$ Not simplified

$6x \neq 0$ ↓ $x \neq 0$

$2x+3 \neq 0$ ↓ $x \neq -\frac{3}{2}$

Domain: $(-\infty,\frac{-3}{2})\cup(\frac{-3}{2},0)\cup(0,\infty)$

7. (5 points each) Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the following functions.

(a) $f(x) = x^2 - 2x + 8$

$$\frac{f(x+h)-f(x)}{h} = \frac{[(x+h)^2 - 2(x+h) + 8] - [x^2 - 2x + 8]}{h}$$

Foil

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h + \cancel{8} - \cancel{x^2} + \cancel{2x} - \cancel{8}}{h}$$

$$= \frac{2xh + h^2 - 2h}{h} = \frac{\cancel{h}(2x+h-2)}{\cancel{h}} = 2x + h - 2$$

(b) $f(x) = \frac{x}{x+4}$

Distribute

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{(x+h)}{(x+h)+4} - \frac{x}{x+4}}{h} = \frac{\frac{x+h}{x+h+4} - \frac{x}{x+4}}{h} \frac{[(x+h+4)(x+4)]}{(x+h+4)(x+4)}$$

$$= \frac{\frac{(x+h)\cancel{(x+h+4)}(x+4)}{\cancel{(x+h+4)}} - \frac{x(x+h+4)\cancel{(x+4)}}{\cancel{(x+4)}}}{h(x+h+4)(x+4)} = \frac{(x+h)(x+4) - x(x+h+4)}{h(x+h+4)(x+4)}$$

$$= \frac{\cancel{x^2} + \cancel{4x} + \cancel{hx} + 4h - \cancel{x^2} - \cancel{hx} - \cancel{4x}}{h(x+h+4)(x+4)}$$

$$= \frac{4\cancel{h}}{\cancel{h}(x+h+4)(x+4)}$$

$$= \frac{4}{(x+h+4)(x+4)}$$

(c) $f(x) = \sqrt{x+6}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h+6} - \sqrt{x+6}}{h}$$

FOIL

$$= \frac{(\sqrt{x+h+6} - \sqrt{x+6})}{h} \frac{(\sqrt{x+h+6} + \sqrt{x+6})}{(\sqrt{x+h+6} - \sqrt{x+6})}$$

$$= \frac{\sqrt{x+h+6}\sqrt{x+h+6} - \cancel{\sqrt{x+h+6}\sqrt{x+6}} + \cancel{\sqrt{x+h+6}\sqrt{x+6}} + \sqrt{x+6}\sqrt{x+6}}{h(\sqrt{x+h+6} - \sqrt{x+6})}$$

$$= \frac{(x+h+6) - (x+6)}{h(\sqrt{x+h+6} - \sqrt{x+6})} = \frac{\cancel{x} + h + \cancel{6} - \cancel{x} - \cancel{6}}{h(\sqrt{x+h+6} - \sqrt{x+6})} = \frac{h}{h(\sqrt{x+h+6} - \sqrt{x+6})} = \frac{1}{\sqrt{x+h+6} - \sqrt{x+6}}$$

8. (5 points each) Find the implied domain of the following functions. Write your answer in <u>interval notation.</u>

(a) $f(x) = \frac{\sqrt{2-7x}}{x+5}$

$2-7x \geq 0$

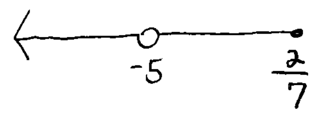
$2 \geq 7x$

$7x \leq 2$

$x \leq \frac{2}{7}$

$x+5 \neq 0$

$x \neq -5$



Domain: $(-\infty,-5)\cup(-5,\frac{2}{7}]$

(b) $f(x) = \sqrt[5]{x-7}$

odd root

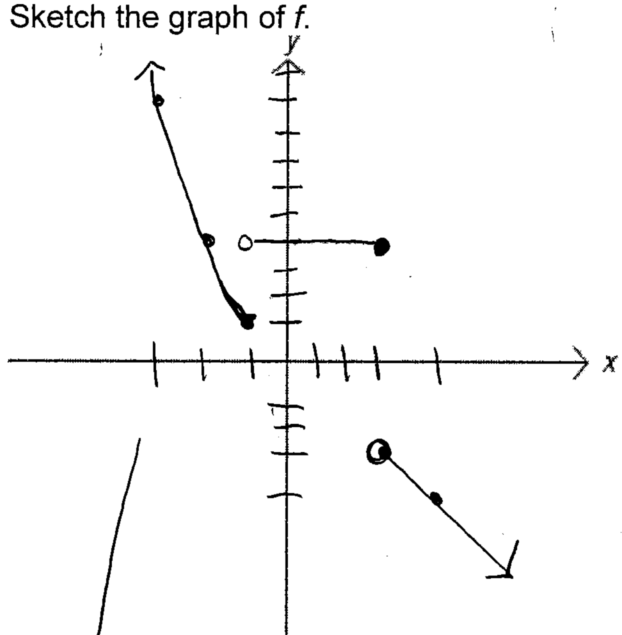
Domain: $(-\infty,\infty)$

9. Let $f(x) = \begin{cases} x^2 & \text{if} \quad x \leq -1 \\ 4 & \text{if} \quad -1 < x \leq 3 \\ -x & \text{if} \quad x > 3 \end{cases}$

(a) Compute the following function values

i. $f(-1) = (-1)^2 = 1$ ii. $f(0) = 4$ iii. $f(4) = -4$

(b) Sketch the graph of *f*.



Piece 1

X	y
-1	1
-2	4
-3	9

Piece 2

x	y
-1	4
0	4
1	4

piece 3

x	y
3	-3
4	-4
5	-5

10. Determine analytically if the following functions are even, odd, or neither even nor odd.

a. $f(x) = 3x^3 + x$ $\quad f(-x) = 3(-x)^3 + (-x)$

$$= -3x^3 - x$$
$$= -(3x^3 + x)$$
$$= -f(x) \quad \text{odd!}$$

b. $g(x) = \frac{3x^4+1}{x^2}$

$$g(-x) = \frac{3(-x)^4+1}{(-x)^2} = \frac{3x^4+1}{x^2} = g(x)$$

even

c. $h(x) = 2x + 4$ $\quad h(-x) = 2(-x) + 4$

$$\underbrace{= -2x + 4}$$

Does not appear to equal $h(x)$ or $-h(x)$.

$(3,10)$ is on graph of h. $(-3,10)$ & $(-3,-10)$ are not.

This means h is neither even nor odd.

11. Suppose (1, -2) is on the graph of $y = f(x)$. Find a point on the graph of the given transformed function.

a. $y = 5f(x) + 2$

$$(1,-2) \xrightarrow[5y]{} (1,-10) \xrightarrow[y+2]{} (1,-8)$$

b. $y = f(-x + 5)$

Method 1

$$\boxed{\begin{aligned} -x+5 &= 1 \\ -x &= -4 \\ x &= 4 \end{aligned}}$$

Method 2

$$\boxed{(1,-2) \xrightarrow[x-5]{} (-4,-2) \xrightarrow[-1x]{} (4,-2)}$$

c. $y = f(\frac{3-x}{2})$

Method 1

$$\boxed{\begin{aligned} \frac{3-x}{2} &= 1 \\ 3-x &= 2 \\ -x &= -1 \\ x &= 1 \end{aligned}}$$

Method 2

$$\boxed{\begin{aligned} &\frac{3-x}{2} = \frac{3}{2} - \frac{1}{2}x \\ &= -\frac{1}{2}x + \frac{3}{2} \\ &(1,-2) \xrightarrow[x-\frac{3}{2}]{} (-\tfrac{1}{2},-2) \xrightarrow[-2x]{} (1,-2) \end{aligned}}$$

$$(1,-2) \rightarrow (1,-2)$$

d. $-f(2x + 5) + 1$

$$2x + 5 = 1$$
$$2x = -4$$
$$x = -2$$

$$(1,-2) \xrightarrow[\frac{x-5}{2}]{} (-2,-2) \xrightarrow[-y+1]{} (-2,3)$$

12. Let $f(x) = \sqrt{x}$. Find a formula for a function *g* whose graph is obtained from *f* from the given sequence of transformations. (1) shift right 1; (2) reflect across the *y*-axis; (3) shift up 2 units

$$g(x) = \sqrt{-x-1} + 2$$

Change 1

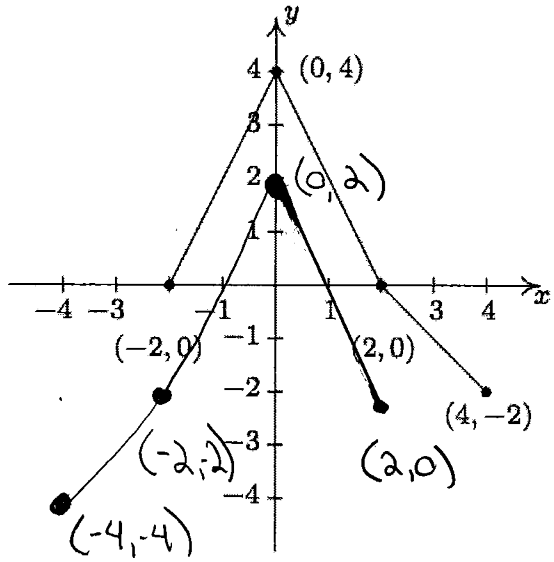
Change 2

Change 3

13. The complete graph of $y = f(x)$ is given below. Graph $f(-x) - 2$

↑ reflect across y-axis

↓ shift Down 2



14. [redacted] A dog walker charges \$9.50 to walk a 30 pound dog and \$13.25 to walk a 45 pound dog.

(a) Find a linear function which models the cost of a dog walk *D* as a function of the weight of the dog *x*. Put your answer in **slope-intercept form**.

x	y
30	9.50
45	13.25

$$m = \frac{13.25 - 9.50}{45 - 30} = \frac{\$3.75}{15 \text{ lb}} = \$.25 \text{ per lb}$$

$D(x) = .25x + b$

$9.50 = .25(30) + b$

$b = 2$

$\boxed{D(x) = .25x + 2}$

(b) According to the model in part (a), how much would the dog walker charge if the dog weighed 80 pounds? Label your answer.

$D(80) = .25(80) + 2$

$= \$22$

(c) Suppose the dog walker charged a dog-owner only \$5. According to the model in part (a), how much does the dog weigh? Label your answer.

$5 = .25x + 2$

$3 = .25x$

$12 = x$

$\boxed{12 \text{ Pounds}}$

15. (5 points) Find the **point-slope form** of the equation of the line through the points (-10, 3) and (1, -8).

$m = \frac{-8-3}{1-(-10)} = \frac{-11}{11} = -1$

$\boxed{y - 3 = -1(x - (-10))}$

16. (5 points) Compute the average rate of change of $f(x) = 2x^2 - x$ over the interval [-1,6].

Average Rate of Change $= \frac{(2(6)^2 - 6) - (2(-1)^2 - (-1))}{6-(-1)} = \frac{66 - 3}{7} = \frac{63}{7} = 9$