Journal of Combinatorial Theory, Series B **83**, 320–330 (2001)
doi:10.1006/jctb.2001.2061, available online at http://www.idealibrary.com on IDEAL®

A Transformation Which Preserves the Clique Number

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Received July 31, 2000; published online July 30, 2001

We introduce a graph transformation which preserves the clique number. When applied to graphs containing no odd hole and no cricket (a particular graph on 5 vertices) the transformation also preserves the chromatic number. Using this transformation we derive a polynomial algorithm for the computation of the clique number of all graphs in a class which strictly contains diamond-free graphs. Furthermore, the transformation leads to a proof that the Strong Perfect Graph Conjecture is true for two new classes of graphs and yields a polynomial time algorithm for the computation of the clique number and the chromatic number for both classes. One of these two classes strictly contains claw-free graphs. © 2001 Elsevier Science

Key Words: clique number; chromatic number; strong perfect graph conjecture.

1. INTRODUCTION

A clique in a graph G is a set of pairwise adjacent vertices. The *clique number* of G, denoted $\omega(G)$, is the size of the largest clique in G. The *chromatic number* of G, denoted $\chi(G)$, is the minimum number of colours needed to colour G so that no two adjacent vertices get the same colour. As usual, P_k and C_k denote respectively a chordless path and a chordless cycle on k vertices. The complement of a graph G is denoted by $\bar{G}$.

A graph is called *perfect* if the vertices of every induced subgraph H can be coloured with $\omega(H)$ colours. Berge [1] introduced perfect graphs and conjectured that a graph is perfect if and only if it does not contain any odd hole or odd antihole. Here a *hole* is a chordless cycle with at least five vertices, and an *antihole* is the complement of a hole. This conjecture is still open and is known as the Strong Perfect Graph Conjecture, SPGC for short. Graphs without any odd holes or odd antiholes are known as *Berge* graphs. A graph G is called *minimal imperfect*, if G itself is not perfect, but every proper induced subgraph of G is perfect. An *even pair* of a graph is any pair of vertices such that every chordless path between them has an even number of edges. Meyniel [7] introduced even pairs and proved that minimal imperfect graphs contain no even pairs.

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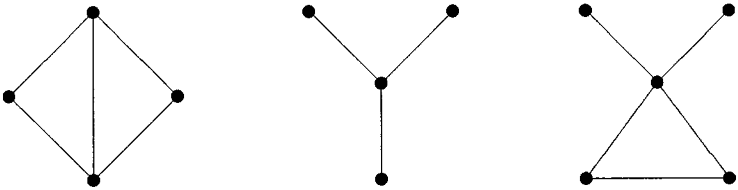


diamond *claw* *cricket*

FIGURE 1

In this paper we generalise results that are known for diamond-free graphs and for claw-free graphs. A *diamond* is the graph obtained by deleting an edge in a clique on four vertices, and a claw is the graph consisting of vertices x, y, z, w and edges xy, xz, xw (see Fig. 1).

We introduce a graph transformation which preserves the clique number. We show that any diamond-free graph G can be transformed into the union of disjoint cliques without modifying the clique number. This leads to a very simple polynomial algorithm for the computation of the clique number in diamond-free graphs. We then extend this result to a class $\mathscr{C}$ of graphs which strictly contains diamond-free graphs. We show that each graph in $\mathscr{C}$ can be transformed into a $(P_5, \overline{P_5}, C_5)$-free graph. Chvátal *et al.* [2] showed that the latter class is perfect, and the authors proposed a linear time algorithm for the maximum clique and the minimum colouring problem.

A *cricket* is the graph obtained from a claw by adding a vertex linked to two adjacent vertices in the claw (see Fig. 1). When the above transformation is applied to a graph which contains no odd hole and no cricket, then we prove that it also preserves the chromatic number. This leads to a proof that the SPGC is true for a class of graphs that strictly contains claw-free graphs.

The paper is organised as follows. We first describe in the next section the graph transformation that preserves the clique number, and we prove that when applied to cricket-free graphs without odd holes, it also preserves the chromatic number. In Section 3, we describe classes of graphs for which the proposed transformation leads to a polynomial algorithm for the computation of the clique number, and we show the validity of the SPGC for two classes of graphs.

2. DESCRIPTION OF THE GRAPH TRANSFORMATION

Let $G = (V, E)$ be a graph with vertex set V and edge set E. Consider any vertex x in V, and let $N(x)$ denote the set of vertices adjacent to x in G.

Let $H_1, ..., H_k$ denote the $k \geqslant 1$ connected components of the subgraph of G induced by the vertices in $N(x)$. We consider the graph transformation which builds a new graph G_x from G as follows. We first remove vertex x from G and then add k new non adjacent vertices $x_1, ..., x_k$, each x_i being linked to all vertices in H_i $(1 \leqslant i \leqslant k)$. The new vertices $x_1, ..., x_k$ are called the *copies* of x.

This transformation increases the number of vertices by $k-1$, but keeps the number of edges constant. If $k=1$, i.e., the neighborhood of vertex x is connected, the transformation does not induce any change in G. In the resulting graph G_x, the neighborhood of each copy x_i of x is connected. Moreover, two copies x_i and x_j of x have no common neighbor in G_x. By construction, the above transformation does not remove any clique from G, and does not create any new one. Hence, G and G_x have the same clique number, i.e., $\omega(G) = \omega(G_x)$.

We can iteratively apply the above transformation as long as there is a vertex whose neighbourhood is disconnected. The resulting graph, denoted $\tilde{G}$ has the same clique number as G. The construction of $\tilde{G}$ is summarised in the following algorithm.

Input: A graph G with vertex set $V = \{v_1, ..., v_n\}$.

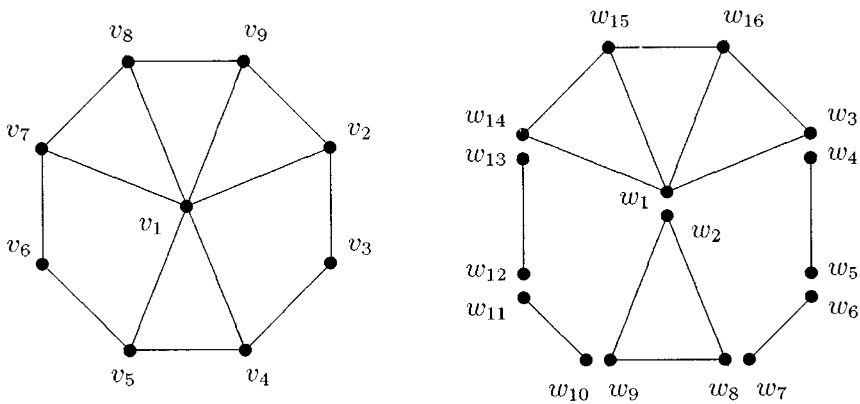
Output: A graph $\tilde{G}$ with vertex set $W = \{w_1, ..., w_s\}$ and such that the neighborhood of each vertex w_i is connected.

(1) Set $s := 0$ and $\tilde{G} := G$;
(2) For $i := 1$ to n do
Let k be the number of connected components of the graph induced by $N(v_i)$;
Set $\tilde{G} := \tilde{G}_{v_i}$ and denote by $w_{s+1}, ..., w_{s+k}$ the copies of v_i; Set $s := s + k$;

This algorithm is illustrated in Fig. 2. Notice that the order of vertex splittings in step (2) is not important and that the output graph $\tilde{G}$ for input graph G is unique. Furthermore, the size of $\tilde{G}$ (number of vertices, number of edges) is polynomial in the size of G. Determining the connected components of a graph $G = (V, E)$ can be done in $O(|V| + |E|)$ [10]. It follows that $\tilde{G}$ can be constructed in $O(|V|^2 + |V|\,|E|)$ time. Moreover, since $\omega(G) = \omega(G_x)$ for all graphs G and any vertex x in G, we have the following property.

Property 1. $\omega(G) = \omega(\tilde{G})$ for all graphs G.

If the graph resulting from this algorithm belongs to a class of graphs for which the clique number can be computed in polynomial time, we have



A graph G The transformed graph $\widetilde{G}$

FIGURE 2

then found a way to determine in polynomial time the clique number of the original graph. We will use this idea in Section 3.

In the following we apply the transformation to determine the chromatic number of a graph. We must however add some restrictions to the input graph G, otherwise the transformation does not necessarily preserve the chromatic number. For example, if G is an odd hole, then $\widetilde{G}$ is the union of disjoint edges, and we therefore have $\chi(G)=3>2=\chi(\widetilde{G})$. Notice also that if a graph G contains no odd hole, then it may happen that G_x contains one. As an illustration, by choosing vertex v_1 in the graph G on the left in Fig. 2, one gets a graph G_{v_1} that contains an odd hole on vertices $w_1, v_2, v_3, v_4, v_5, v_6, v_7$, while G does not contain any odd hole.

LEMMA 1. *Let G be a cricket free graph without odd holes, and let x be any vertex in G. Then each pair of copies x_i, x_j of x in G_x is an even pair.*

Proof. Suppose there are two copies x_i and x_j of x in G_x which are linked by a chordless path $P=(x_i=z_0, z_1, ..., z_{2k+1}=x_j)$ having an odd number of edges. We can assume that P is the shortest such odd-length chordless path between two copies of x in G_x. None of the vertices z_1 to z_{2k} is a copy of x, since this would contradict the minimality of P. Hence, vertices $z_1, ..., z_{2k}$ induce a chordless path in G. Clearly, x is adjacent to z_1 and z_{2k} in G. Now, x is not adjacent to z_2 in G, otherwise z_1 and z_2 would be in the same connected component of $N(x)$ and x_i would therefore be adjacent to z_2 in G_x. Similarly, x is not adjacent to z_{2k-1} in G.

Finally, if G contains no induced triangle xz_iz_{i+1} $(3\leqslant i\leqslant 2k-3)$, then G contains an odd hole, and if G contains such a triangle xz_iz_{i+1}, then vertices $x, z_1, z_i, z_{i+1}, z_{2k}$ induce a cricket in G, a contradiction. ∎

The proof of the next lemma uses the standard proof technique which can be found in [3, 7].

LEMMA 2. *Let G be a cricket free graph without odd holes, and let x be any vertex in G. Then,* $\chi(G)=\chi(G_x)$.

Proof. Consider first any colouring of G in $\chi(G)$ colours. One can also colour G_x with $\chi(G)$ colours by assigning to each copy of x in G_x the same colour as x in G, and by giving to all other vertices the same colour as in G. Hence, $\chi(G) \geqslant \chi(G_x)$.

Consider now any colouring of G_x in $\chi(G_x)$ colours. If all copies of x have the same colour, one can also colour G with $\chi(G_x)$ colours by assigning to x the same colour as its copies in G_x, and by giving to all other vertices the same colour as in G_x. Assume now that two copies x_i and x_j of x have different colours r and s, respectively. Consider the subgraph G_{rs} of G_x containing only those vertices having colour r or s. Let C_{rs} be the connected component of G_{rs} containing vertex x_i. It follows from Lemma 1 that no copy of x with colour s belongs to C_{rs}. Hence, by exchanging colours r and s in C_{rs} one gets a new colouring of G_x in $\chi(G_x)$ colours, and we have strictly increased the number of copies of x having colour s. By applying this colour exchange procedure iteratively, one can assign colour s to all copies of x, and we have seen above that G can then easily be coloured in $\chi(G_x)$ colours. Hence, $\chi(G) \leqslant \chi(G_x)$. ∎

The *diameter* $d(G)$ of a graph G is defined as the length of the longest shortest path between two vertices in G.

LEMMA 3. *Let G be an H-free graph, where H is a graph with diameter* $d(H) \leqslant 2$, *and let x be any vertex in G. Then* G_x *is also H-free.*

Proof. Remember first that the copies of x in G_x are non-adjacent and have no common neighbour. Suppose there is a set $\{h_1, ..., h_k\}$ of vertices which induces an H in G_x. If $\{h_1, ..., h_k\}$ contains no copy of x, then $\{h_1, ..., h_k\}$ also induces an H in G, a contradiction. If $\{h_1, ..., h_k\}$ contains two copies of x, then these copies are non adjacent, and since $d(H) \leqslant 2$, they have a common neighbour, a contradiction.

So assume $\{h_1, ..., h_k\}$ contains exactly one copy of x, say h_1. Then, all neighbours of h_1 in $\{h_2, ..., h_k\}$ are also neighbours of x in G. Since G is H-free, there must exist a vertex h_i in $\{h_2, ..., h_k\}$ which is adjacent to x in G but not to h_1 in G_x. Since $d(H) \leqslant 2$, h_1 and h_i have a common neighbour h_j. Hence, x is also adjacent to h_j, which means that h_i and h_j belong to the same connected component, of $N(x)$ in G. So, h_1 is either linked to both h_i and h_j, or to none of them, a contradiction. ∎

LEMMA 4. *Let G be a cricket-free graph without odd holes, and let x be any vertex in G. Then G_x is also cricket-free and without odd holes.*

Proof. Since the diameter of a cricket is 2, it follows from Lemma 3 that G_x is also cricket-free. Now suppose that G_x contains an odd hole $C=(c_1, ..., c_{2k+1})$ $(k \geqslant 2)$. At least one vertex in C must be a copy of x, otherwise G also contains an odd hole. Moreover, C cannot contain more than one copy of x, otherwise there is a chordless path with an odd number of edges between two copies of x, which contradicts Lemma 1.

Hence, C contains exactly one copy of x, say c_1. Vertex x is therefore adjacent to c_2 and to c_{2k+1} in G. It follows that x is not adjacent to c_3 in G, else c_2 and c_3 would belong to the same connected component of $N(x)$, which would imply that c_1 is adjacent to c_2 and c_3, or to none of them, a contradiction. Similarly, x is not adjacent to c_{2k} in G.

Notice that x must be adjacent to two consecutive vertices c_i and c_{i+1} on C, else the subgraph of G induced by $x, c_2, c_3, ..., c_{2k+1}$ contains an odd hole. But this means that vertices $x, c_2, c_i, c_{i+1}, c_{2k+1}$ induce a cricket in G, a contradiction. ∎

COROLLARY 1. *Let G be a cricket free graph without odd holes. Then $\chi(G)=\chi(\tilde{G})$.*

Proof. This is a direct consequence of Lemma 2 and Lemma 4.

COROLLARY 2. *Let $G=(V, E)$ be a cricket free graph without odd holes. Given a colouring of $\tilde{G}$ in $\chi(\tilde{G})=\chi(G)$ colours, one can colour G with the same number of colours in $O(|V|\,|E|)$ time.*

Proof. It follows from Lemma 2 that given any vertex x in a cricket-free graph $G=(V, E)$ without odd holes, and any colouring of G_x in $\chi(G_x)=\chi(G)$ colours, one can colour G with the same number of colours by performing bichromatic exchanges until all copies of x get the same colour. This takes $O(|E|)$ time. Since $\tilde{G}$ is obtained from G after $|V|$ successive transformations, a colouring of G in $\chi(G)$ colours can be derived from a colouring of $\tilde{G}$ in $\chi(G)$ colours in $O(|V|\,|E|)$ time. ∎

COROLLARY 3. *Let G be a cricket-free Berge graph. Then $\tilde{G}$ is also Berge.*

Proof. Let G be a cricket-free Berge graph. Since the diameter of an antihole is 2, it follows from Lemma 3 that $\tilde{G}$ has no odd antihole. Moreover, we know from Lemma 4 that $\tilde{G}$ has no odd hole. Hence, $\tilde{G}$ is Berge. ∎

3. SPECIAL CLASSES OF GRAPHS

The clique number of diamond-free graphs can easily be computed by means of the transformation described in the previous section. Indeed, in such a case, the transformed graph $\tilde{G}$ has a very special structure as stated in the following Lemma.

LEMMA 5. *Let G be a diamond-free graph. Then $\tilde{G}$ is the union of disjoint cliques.*

Proof. Since the diameter of a diamond is 2, it follows from Lemma 3 that $\tilde{G}$ is also diamond-free. Suppose now that $\tilde{G}$ contains a chordless path $P=(v_1, v_2, v_3)$. Since the neighbourhood of vertex v_2 in $\tilde{G}$ must be connected, there exists a chordless path $(v_1=p_1, p_2, ..., p_k=v_3)$ in the neighbourhood of v_2. Hence, vertices v_2, p_1, p_2, p_3 induce a diamond in $\tilde{G}$, a contradiction. ∎

Since $\omega(G)=\omega(\tilde{G})$ for all graphs $G=(V, E)$, we can compute the clique number of a diamond-free graph by first constructing $\tilde{G}$, and then determining the largest connected component in $\tilde{G}$. This leads to an $O(|V|^2+|V|\,|E|)$ time algorithm. We now extend this result to a larger class of graphs. Consider the two graphs depicted in Fig. 3. They both contain a diamond as subgraph. The class of (gem, F_1)-free graphs therefore strictly contains all diamond-free graphs.

LEMMA 6. *Let G be a (gem, F_1)-free graph. Then, $\tilde{G}$ is (P_5, $\overline{P_5}$, C_5)-free.*

Proof. Since the diameter of both forbidden subgraphs in G is 2, it follows from Lemma 3 that G is also (gem, F_1)-free. Moreover, the neighbourhood $N(x)$ of each vertex x in $\tilde{G}$ is connected and P_4-free, else the subgraph induced by $\{x\} \cup N(x)$ contains a gem.

Suppose that $\tilde{G}$ contains an induced $P_5=(x_1, x_2, x_3, x_4, x_5)$, or an induced $C_5=(x_1, x_2, x_3, x_4, x_5)$. Since $N(x_3)$ is connected and P_4-free,

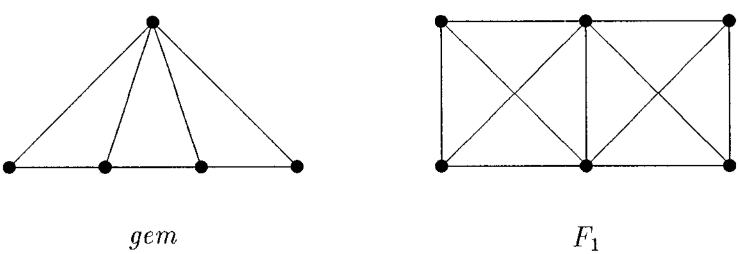


FIGURE 3

there exists a vertex y_1 which is adjacent to x_2, x_3 and x_4 in $\tilde{G}$. Vertex y_1 is neither adjacent to x_1, nor to x_5 else vertices x_1, x_2, x_3, x_4, y_1 or x_2, x_3, x_4, x_5, y_1 induce a gem in $\tilde{G}$. Since $N(x_2)$ is also connected and P_4-free, there exists a vertex y_2 which is adjacent to x_1, x_2, x_3 and y_1 in $\tilde{G}$. Vertex y_2 is not adjacent to x_4, else vertices x_1, x_2, x_3, x_4, y_2 induce a gem in $\tilde{G}$. By symmetry, there exists a vertex y_3 which is adjacent to x_3, x_4, x_5, y_1 but not to x_2 in $\tilde{G}$. Also, vertices y_2 and y_3 are not adjacent, else x_2, x_4, y_1, y_2, y_3 induce a gem in $\tilde{G}$. But now, vertices x_2, x_3, x_4, y_1, y_2, y_3 induce an F_1 in $\tilde{G}$, a contradiction.

Suppose now that the complement $\overline{\tilde{G}}$ of $\tilde{G}$ contains an induced $P_5 = (x_1, x_2, x_3, x_4, x_5)$. Since $N(x_5)$ is connected and P_4-free in $\tilde{G}$, there exists a vertex y_1 which is adjacent to x_1, x_2, x_3 and x_5 in $\tilde{G}$. Vertex y_1 is not adjacent to x_4 else vertices x_2, x_3, x_4, x_5, y_1 induce a gem in $\tilde{G}$. Since $N(x_2)$ is also connected and P_4-free in $\tilde{G}$, there exists a vertex y_2 which is adjacent to x_2, x_4, x_5 and y_1. Vertex y_2 is neither adjacent to x_3 nor to x_1 else x_2, x_3, x_4, x_5, y_2 or x_1, x_3, x_4, y_1, y_2 induce a gem in $\tilde{G}$. But now, vertices $x_1, x_2, x_3, x_5, y_1, y_2$ induce an F_1 in $\tilde{G}$, a contradiction. ∎

THEOREM 1. *The clique number of a (gem, F_1)-free graph $G = (V, E)$ can be computed in $O(|V|^2 + |V|\,|E|)$ time.*

Proof. We know from Property 1 that $\omega(G) = \omega(\tilde{G})$ and it follows from Lemma 6 that $\tilde{G}$ is $(P_5, \overline{P_5}, C_5)$-free. It is now sufficient to observe that the construction of $\tilde{G}$ takes $O(|V|^2 + |V|\,|E|)$ time, while the computation of the clique number of a $(P_5, \overline{P_5}, C_5)$-free graph can be performed in $O(|V| + |E|)$ time [2]. ∎

We now prove the validity of the SPGC for (cricket, gem, F_1)-free graphs. Notice that a gem-free graph does not contain any antihole of length larger than 6. Moreover, the antihole and the hole on 5 vertices are isomorphic. Hence, a graph is Berge and (cricket, gem, F_1)-free if and only if it is (cricket, gem, F_1)-free and without odd holes.

THEOREM 2. *All (cricket, gem, F_1)-free graphs without odd holes are perfect.*

Proof. We know from Corollary 1 that if G is cricket-free and without odd holes, then $\chi(G) = \chi(\tilde{G})$. Also, it follows from Lemma 6 that if G is (gem, F_1)-free, then $\tilde{G}$ is $(P_5, \overline{P_5}, C_5)$-free. Since these graphs are perfect [2], we can conclude that if G is without odd holes and (cricket, gem, F_1)-free, then $\omega(G) = \omega(\tilde{G}) = \chi(\tilde{G}) = \chi(G)$. ∎

THEOREM 3. *A colouring in* $\chi(G)$ *colours of a (cricket, gem,* F_1*)-free graph* $G=(V, E)$ *without odd holes can be determined in* $O(|V|^2+|V|\,|E|)$ *time.*

Proof. The construction of $\tilde{G}$ from G requires $O(|V|^2+|V|\,|E|)$ time. Since $\tilde{G}$ is $(P_5, \overline{P_5}, C_5)$-free, it can be coloured in $\chi(\tilde{G})=\chi(G)$ colours in $O(|V|+|E|)$ time [2]. Finally, we know from Corollary 2 that a colouring of G in $\chi(G)$ colours can be derived from the colouring of $\tilde{G}$ in $O(|V|\,|E|)$ time. ▮

Claw-free Berge graphs have been well studied, and several proofs of their perfectness have been given [4, 6, 8]. Moreover, Hsu [5] has proposed an $O(|V|^4)$ algorithm which determines the chromatic number of claw-free Berge graphs. In the following, we prove the validity of the Strong Perfect Graph Conjecture for a class of graphs that strictly contains claw-free graphs. Consider the three graphs depicted in Fig. 4. Notice that the cricket, F_2, F_3, and F_4 contain a claw.

LEMMA 7. *Let* G *be a* (F_2, F_3, F_4)*-free graph. Then* $\tilde{G}$ *is claw free.*

Proof. Since the diameter of all three forbidden subgraphs is 2, we know from Lemma 3 that $\tilde{G}$ is (F_2, F_3, F_4)-free. Now suppose that $\tilde{G}$ contains an induced claw on vertices c_0, c_1, c_2, c_3, where c_0 is adjacent to the three other vertices. Since the neighbourhood of each vertex in $\tilde{G}$ is connected, there exists a chordless path $(c_1=p_0, ..., p_k=c_2)$ from c_1 to c_2 in $N(c_0)$, as well as a chordless path $(c_2=q_0, ..., q_l=c_3)$ from c_2 to c_3 in $N(c_0)$. Since F_4 is a forbidden subgraph, both paths are of length 2 or 3. Assume first that both paths are of length 2 (i.e., $k=l=2$). Vertex p_1 is not adjacent to c_3, else $\tilde{G}$ contains an F_2. For the same reason, vertex q_1 is not adjacent to c_1. Hence, we now have either an F_3 if p_1 is adjacent to q_1, or an F_4 otherwise, a contradiction.

Therefore, among the three chordless paths linking c_1 to c_2, c_1 to c_3 and c_2 to c_3 in $N(c_0)$, at least two are of length 3. We may assume that there exist chordless paths $(c_1=p_0, p_1, p_2, p_3=c_2)$ and $(c_2=q_0, q_1, q_2, q_3=c_3)$

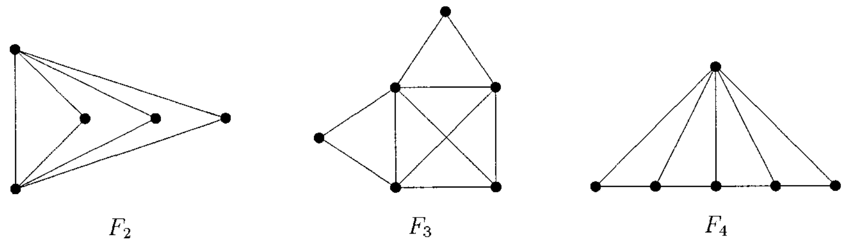


FIGURE 4

in $N(c_0)$. Vertex c_3 cannot be adjacent to both p_1 and p_2, else vertices c_0, c_1, c_2, c_3, p_1, p_2 induce an F_3 in $\tilde{G}$. Also, vertex c_3 cannot be adjacent to exactly one vertex among p_1 and p_2, say p_1, else vertices c_0, c_1, c_3, p_1, p_2 induce an F_2 in $\tilde{G}$. Hence, c_3 is neither adjacent to p_1, nor to p_2. By symmetry, q_2 is not adjacent to c_1. Notice that vertices c_0, c_1, c_2, q_2 also induce a claw, which means that we also know that q_2 is neither adjacent to p_1, nor to p_2. But now, vertices c_0, c_2, p_1, q_2 induce a claw, while the paths (p_1, p_2, c_2) and (c_2, q_1, q_2) in $N(c_0)$ are of length 2, a contradiction. ∎

THEOREM 4. *All (cricket, F_2, F_3, F_4)-free Berge graphs are perfect.*

Proof. Let G be a (cricket,F_2, F_3, F_4)-free Berge graph. It follows from Corollary 3 and Lemma 7 that $\tilde{G}$ is Berge and claw-free. Also, it follows from Corollary 1 that $\chi(G)=\chi(\tilde{G})$. Since claw-free Berge graphs are known to be perfect [8], we conclude that $\omega(G)=\omega(\tilde{G})=\chi(\tilde{G})=\chi(G)$. ∎

THEOREM 5. *A colouring of a (cricket, F_2, F_3, F_4)-free Berge graph $G=(V, E)$ in $\chi(G)$ colours can be determined in $O(|V|^4)$ time.*

Proof. The construction of $\tilde{G}$ from G requires $O(|V|^2+|V|\,|E|)$ time. Hsu [5] has shown how to colour $\tilde{G}$ in $\chi(\tilde{G})=\chi(G)$ colours in $O(|V|^4)$ time. Finally, we know from Corollary 2 that a colouring of G in $\chi(G)$ colours can be derived from the colouring of $\tilde{G}$ in $O(|V|\,|E|)$ time. ∎

Theorem 4 proves the validity of the SPGC for a class of graphs that strictly contains claw-free graphs. A similar result has been obtained by Sun who has shown the validity of the SPGC for dart-free graphs, where a *dart* is the graph obtained from a claw by adding a vertex linked to all vertices in the claw, except one of degree 1 [9].

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