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Calculus

Previous year Questions from 2020 to 1992

2021-22

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2020

1. Evaluate $\lim\limits_{x\to\frac{\pi}{4}}(\tan x)^{\tan 2x}$. **[10 Marks]**

2. Find all the asymptotes of the curve $(2x+3)y=(x-1)^2$ **[10 Marks]**

3. Evaluate $\int_0^1 \tan^{-1}\left(1-\frac{1}{x}\right)dx.$ **[15 Marks]**

4. Consider the function $f(x)=\int_0^x (t^2-5t+4)(t^2-5t+6)dt$
   (i) Find the critical points of the function $f(x)$
   (ii) Find the points at which local minimum occurs.
   (iii) Find the points at which local maximum occurs.
   (iv) Find the number of zeros of the function $f(x)$ in $[0,5]$ **[20 Marks]**

5. Find an extreme value of the function $u=x^2+y^2+z^2$ subject to the condition $2x+3y+5z=30$ by using Lagrange's method of undetermined multiplier. **[20 Marks]**

2019

6. Let $f:\left[0,\frac{\pi}{2}\right]\to R$ be a continuous function such that $f(x)=\frac{\cos^2 x}{4x^2-\pi^2},\quad 0\le x\le\frac{\pi}{2}$. Find the value of $f\left(\frac{\pi}{2}\right)$ **[10 Marks]**

7. Let $f:D(\subseteq R^2)\to R$ be a function and $(a,b)\in D$. If $f(x,y)$ is continuous at (a,b), then show the functions $f(x,b)$ and $f(a,y)$ are continuous at $x=a$ and at $y=b$ respectively. **[10 Marks]**

8. Is $f(x)=|\cos x|+|\sin x|$ differentiable at $x=\frac{\pi}{2}$? If yes, then find its derivative at $x=\frac{\pi}{2}$ $x=\frac{\pi}{2}$ If no, then a proof of it. **[15 Marks]**

9. Find the maximum and the minimum value of the function $f(x)=2x^3-9x^2+12x+6$ on the interval $[2,3]$ **[10 Marks]**

10. If $u=\sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then show that $\sin^2 u$ is a homogeneous function of x and y of degree $-\frac{1}{6}$ hence show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right)$ **[12 Marks]**

11. Using the Jacobian method, show that if $f'(x)=\frac{1}{1+x^2}$ and $f(0)=0$ then $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$ **[8 Marks]**

2018

12. Determine if $\lim\limits_{z\to 1}(1-z)\tan\frac{\pi z}{2}$ exists or not. If the limit exists, then find its value. **[10 Marks]**

13. Find the limit $\lim_{n\to\infty}\frac{1}{n^2}\sum_{r=0}^{n-1}\sqrt{n^2-r^2}$. **[10 Marks]**

14. Find the shortest distance from the point $(1,0)$ to the parabola $y^2=4x$ **[13 Marks]**

15. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolves about the x-axis. Find the volume of the solid of revolution. **[13 Marks]**

16. Let $f(x,y)=\begin{cases} xy^2, & y>0 \\ -xy^2, & y\le 0\end{cases}$. Determine which of $\frac{\partial f}{\partial x}(0,1), \frac{\partial f}{\partial y}(0,1)$ and exists and which does not exist. **[12 Marks]**

17. Find the maximum and the minimum values of x^4-5x^2+4 on the interval $[2,3]$. **[13 Marks]**

18. Evaluate the integral $\int_0^a\int_{x/a}^{x}\frac{xdydx}{x^2+y^2}$ **[12 Marks]**

2017

19. Integrate the function $f(x,y)=xy(x^2+y^2)$ over the domain $R:\{-3\le x^2-y^2\le 3, 1\le xy\le 4\}$ **[10 Marks]**

20. Find the volume of the solid above the xy-plane and directly below the portion of the elliptic paraboloid $x^2+\frac{y^2}{4}=z$ which is cut off by the plane $z=9$ **[15 Marks]**

21. If $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\ne(0,0) \\ 0 & , (x,y)=(0,0)\end{cases}$

calculate $\frac{\partial^2 f}{\partial x\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$ at $(0,0)$. **[15 Marks]**

22. Examine if the improper integral $\int_0^3\frac{2xdx}{(1-x^2)^{2/3}}$, exists. **[10 Marks]**

23. Prove that $\frac{\pi}{3}\le\iint_D\frac{dxdy}{\sqrt{x^2+(y-2)^2}}\le\pi$ where D is the unit disc. **[10 Marks]**

2016

24. Evaluate: $I=\int_0^1\sqrt[3]{x\log\left(\frac{1}{x}\right)}\,dx$ **[10 marks]**

25. Find the matrix and minimum values of $x^2+y^2+z^2$ subject to the conditions $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and $x+y-z=0$ **[20 marks]**

26. Let $f(x,y)=\begin{cases}\frac{2x^4-5x^2y^2+y^5}{(x^2+y^2)^2}, & x,y)\ne(0,0) \\ 0 & , x,y)=(0,0)\end{cases}$ find a $\delta>0$ such that $|f(x,y)-f(0,0)|<0.01$ whenever $\sqrt{x^2+y^2}<\delta$ **[15 marks]**

27. Find the surface area of the plane $x+2y+2z=12$ cut off by $x=0, y=0$ and $x^2+y^2=16$ **[15 marks]**

28. Evaluate $\iint_R f(x,y)dxdy$, over the rectangle $R=[0,1;0,1]$ where $f(x,y)=\begin{cases} x+y, & \text{if } x^2 < y < 2x^2 \\ 0, & \text{elsewhere} \end{cases}$ **[15 marks]**

2015

29. Evaluate the following limit $\lim_{x\to a}\left(2-\frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ **[10 Marks]**

30. Evaluate the following integral: $\int_{\pi/6}^{\pi/2}\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$ **[10 Marks]**

31. A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base. **[13 Marks]**

32. Which point of the sphere $x^2+y^2+z^2=1$ is at the maximum distance from the point $(2,1,3)$ **[13 Marks]**

33. Evaluate the integral $\iint_R (x-y)^2\cos^2(x+y)\,dxdy$ where R is the rhombus with successive vertices as $(\pi,0),(2\pi,\pi),(\pi,2\pi),(0,\pi)$ **[12 Marks]**

34. Evaluate $\iint_R \sqrt{|y-x^2|}\,dxdy$ where $R=[-1,1;0,2]$ **[13 Marks]**

35. For the function $f(x,y)=\begin{cases} \dfrac{x^2-x\sqrt{y}}{x^2+y}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ Examine the continuity and differentiability. **[12 Marks]**

2014

36. Prove that between two real roots $e^x\cos x+1=0$, a real root of $e^x\sin x+1=0$ lies. **[10 Marks]**

37. Evaluate: $\int_0^1\frac{\log_e(1+x)}{1+x^2}dx$. **[10 Marks]**

38. By using the transformation $x+y=u, y=uv$ evaluate the integral $\iint\{xy(1-x-y)\}^{\frac{1}{2}}\,dxdy$ taken over the area enclosed by the straight lines $x=0, y=0$ and $x+y=1$. **[15 Marks]**

39. Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. **[15 Marks]**

40. Find the maximum or minimum values of $x^2+y^2+z^2$ subject to the condition $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$ interpret result geometrically **[20 Marks]**

2013

41. Evaluate $\int_0^1\left(2x\sin\frac{1}{x}-cos\frac{1}{x}dx\right)$ **[10 Marks]**

42. Using Lagrange's multiplier method find the shortest distance between the line $y=10-2x$ and the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ **[20 Marks]**

43. Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function $f(x,y)=\begin{cases}\dfrac{xy^3}{x+y^2}, & (x,y)\neq(0,0)\\ 0 & , (x,y)=(0,0)\end{cases}$

Also distance the continuity of f_{xy} and f_{yx} at $(0,0)$. **[15 Marks]**

44. Evaluate $\iint_D xydA$ where D is the region bounded by the line $y=x-1$ and the parabola $y^2=2x+6$.

[15 Marks]

2012

45. Define a function f of two real variables in the plane by $f(x,y)=\begin{cases}\dfrac{x^3\cos\frac{1}{y}+y^3\cos\frac{1}{x}}{x^2+y^2} & \text{for } x,y\neq 0\\ 0, \text{otherewis}\end{cases}$

Check the continuity and differentiability of f at $(0,0)$. **[12 Marks]**

46. Let p and q be positive real numbers such that $\frac{1}{p}+\frac{1}{q}=1$ show that for real numbers $ab\geq 0$

$ab\frac{a^p}{p}+\frac{b^q}{q}$. **[12 Marks]**

47. Find the point of local extreme and saddle points of the function f for two variables defined by $f(x,y)=x^3+y^3-63(x+y)+12xy$ **[20 Marks]**

48. Defined a sequence s_n of real numbers by $s_n=\sum_{i=1}^{n}\frac{(\log(n+i)-\log n)^2}{n+1}$ does $\lim_{n\to\infty} s_n$ exist? If so compute the value of this limit and justify your answer **[20 Marks]**

49. Find all the real values of p and q so that the integral $\int_0^1 x^p\left(\log\frac{1}{x}\right)^q dx$ converges **[20 Marks]**

2011

50. Find $\lim_{(x,y)\to(0,0)}\frac{x^2y}{x^3+y^3}$ if it exists **[10 Marks]**

51. Let f be a function defined on $\mathbb{R}$ such that $f(0)=-3$ and $f'(x)\leq 5$ for all values of x in $\mathbb{R}$ How large can $f(2)$ possibly be? **[10 Marks]**

52. Evaluate:

(i) $\lim_{x\to 2} f(x)$ Where $f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & , x\neq 2\\ \pi & , x=2\end{cases}$ (ii) $\int_0^1 \ell nxdx$. **[20 Marks]**

2010

53. A twice differentiable function $f(x)$ is such that $f(a)=0=f(b)$ and $f(c)>0$ for $a<c<b$ prove that there be is at least one point $\xi, a<\xi<b$ for which $f''(\xi)<0$ **[12 Marks]**

54. Dose the integral $\int_{-1}^{1}\sqrt{\frac{1+x}{1-x}}$ exist if so, find its value **[12 Marks]**

55. Show that a box (rectangular parallelepiped) of maximum volume V with prescribed surface area is a cube. **[20 Marks]**

56. Let D be the region determine by the inequalities $x>0, y>0, z<8$ and $z>x^2+y^2$ compute $\iiint_D 2xdxdydz$. **[20 Marks]**

57. If $f(x,y)$ is a homogeneous function of degree n in x and y, and has continuous first and second order partial derivatives then show that

$(i)\ x\frac{\partial^2 f}{\partial x}+y\frac{\partial^2 f}{\partial y}=nf$ $\quad (ii)\ x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial\times\partial y}+y^2\frac{\partial^2 f}{\partial y^2}=n(n-1)f$ **[20 Marks]**

2009

58. Suppose the f'' is continuous on $[1,2]$ and that f has three zeroes in the interval $(1,2)$ show that f'' has least one zero in the interval $(1,2)$. **[12 Marks]**

59. If f is the derivative of same function defined on $[a,b]$ prove that there exists a number $\eta\in[a,b]$ such that $\int_a^b f(t)dt=f(\eta)(b-a)$ **[12 Marks]**

60. If $x=3\pm 0.01$ and $y=4\pm 0.01$ with approximately what accuracy can you calculate the polar coordinate r and θ of the point $P(x,y)$ Express you estimates as percentage changes of the value that r and θ have at the point $(3,4)$ **[20 Marks]**

61. A space probe in the shape of the ellipsoid $4x^2+y^2+4z^2=16$ enters the earth atmosphere and its surface beings to heat. After one hour, the temperature at the point (x,y,z) on the probe surface is given by $T(x,y,z)=8x^2+4yz-16z+1600$ Find the hottest point on the probe surface. **[20 Marks]**

62. Evaluate $I=\iint_s xdydz+dzdx+xz^2dxdy$ where S is the outer side of the part of the sphere $x^2+y^2+z^2=1$ in the first octant. **[20 Marks]**

2008

63. Find the value of $\lim_{x\to 1}\ln(1-x)\cot\frac{\pi x}{2}$. **[12 Marks]**

64. Evaluate $\int_0^1 (x\ln x)^3dx$. **[12 Marks]**

65. Determine the maximum and minimum distances of the origin from the curve given by the equation $3x^2+4xy+6y^2=140$. **[20 Marks]**

66. Evaluate the double integral $\int_y^a \frac{xdxdy}{x^2+y^2}$ by changing the order of integration **[20 Marks]**

67. Obtain the volume bounded by the elliptic paraboloid given by the equations $z-x^2+9y^2 \,\&\, z=18-x^2-9y^2$ **[20 Marks]**

2007

68. Let $f(x),(x\in(-\pi,\pi))$ be defined by $f(x)=\sin|x|$ is f continuous on $(-\pi,\pi)$ if it is continuous then is it differentiable on $(-\pi,\pi)$? **[12 Marks]**

69. A figure bounded by one arch of a cycloid $x=a(t-\sin t),\, y=a(1-\cos t),\, t\in[0,2\pi]$ and the x-axis is revolved about the x-axis. Find the volume of the solid of revolution **[12 Marks]**

70. Fin a rectangular parallelepiped of greatest volume for a give total surface area S using Lagrange's method of multipliers **[20 Marks]**

71. Prove that if $z=\phi(y+ax)+\psi(y-ax)$ then $a^2\frac{\partial^2 z}{\partial y^2}-\frac{\partial^2 z}{\partial x^2}=0$ for any twice differentiable φ and ψ is a constant. **[15 Marks]**

72. Show that $e^{-x}x^n$ is bounded on $[0,\infty)$ for all positive integral values of n. Using this result show that $\int_0^\infty e^{-x}x^n dx$ exists. **[25 Marks]**

2006

73. Find a and b so that $f'(2)$ exists where $f(x)=\begin{cases}\frac{1}{|x|}, & \text{if } |x|>2\\ a+bx^2 & \text{if } |x|\le 2\end{cases}$ **[12 Marks]**

74. Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma function and hence evaluate the integral $\int_0^1 x^6\sqrt{(1-x^2)}dx$ **[12 Marks]**

75. Find the values of a and b such that $\lim_{x\to 0}\frac{a\sin^2 x\times b\log\cos x}{x^4}=\frac{1}{2}$. **[15 Marks]**

76. If $z=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$ show that $x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0$. **[15 Marks]**

77. Change the order of integration in $\int_x^\infty \frac{e^{-y}}{y}dydx$ and hence evaluate it. **[15 Marks]**

78. Find the volume of the uniform ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **[15 Marks]**

2005

79. Show that the function given below is not continuous at the origin $f(x,y)=\begin{cases}0 \text{ if } xy=0\\ 1 \text{ if } xy\ne 0\end{cases}$ **[12 Marks]**

80. Let $R^2\to R$ be defined as $f(x,y)=\frac{xy}{\sqrt{(x^2+y^2)}},(x,y)\ne(0,0),\ f(0,0)=0$ prove that f_x and f_y exist at $(0,0)$ but f is not differentiable at $(0,0)$. **[12 Marks]**

81. If $u = x+y+z, uv = y+z$ and $uvw = z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ **[15 Marks]**

82. Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ in terms of Beta function. **[15 Marks]**

83. Evaluate $\iiint_v zdV$ where V the volume is bounded below by the cone $x^2+y^2=z^2$ and above by the sphere $x^2+y^2+z^2=1$ lying on the positive side of the y-axis. **[15 Marks]**

84. Find the x-coordinate of the center of gravity of the solid lying inside the cylinder $x^2+y^2=2ax$ between the plane $z=0$ and the paraboloid $x^2+y^2=az$. **[15 Marks]**

2004

85. Prove that the function f defined on $[0,4]$ $f(x)=[x]$ greatest integer $\le x, x\in[0,4]$ is integrable on $[0,4]$ and that $\int_0^4 f(x)dx = 6$. **[12 Marks]**

86. Shaw that $x-\frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$ $x>0$.

[12 Marks]

87. Let the roots of the equation in $\lambda(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ be u,v,w proving that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = -2\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$. **[15 Marks]**

88. Prove that an equation of the form $x^n=\alpha$ where $n\in N$ and $\alpha>0$ is a real number has a positive root.

[15 Marks]

89. Prove that $\int \frac{x^2+y^2}{p}dx = \frac{\pi ab}{4}[4+(a^2+b^2)(a^{-2}+b^{-2})]$ when the integral is taken round the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and p is three length of three perpendicular from the center to the tangent. **[15 Marks]**

90. If the function f is defined by $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$ then show that possesses both the partial derivative at but it is not continuous thereat. **[15 Marks]**

2003

91. Let f be a real function defined as follow:
$f(x)=x, -\le x<1$
$f(x+2)=x, \forall x\in R$
Show that f is discontinuous at every odd integer. **[12 Marks]**

92. For all real numbers x, $f(x)$ is given as $f(x)=\begin{cases}e^x+a\sin x, & x<0\\ b(x-1)^2+x-2, & x\ge 0\end{cases}$. Find values of a and b for which is differentiable at $x=0$. **[12 Marks]**

93. A rectangular box open at the top is to have a volume of $4m^3$. Using Lagrange's method of multipliers find the dimension of the box so that the material of a given type required to construct it may be least. **[15 Marks]**

94. Test the convergent of the integrals(i) $\int_0^1 \frac{dx}{x^{\frac{1}{3}}\left(1+x^2\right)}$ (ii) $\int_0^\infty \frac{\sin^2 x}{x^2}dx$ **[15 Marks]**

95. Evaluate the integral $\int_0^a \int_{\frac{y^2}{a}}^{y} \frac{ydxdy}{(a-x)\sqrt{ax-y^2}}$ **[15 Marks]**

96. Find the volume generated by revolving by the real bounded by the curves $(x^2+4a^2)y=8a^3$, $2y=x$ and $x=0$ about the y-axis. **[15 Marks]**

2002

97. Show that $\frac{b-a}{\sqrt{1-a^2}} \le \sin^{-1} b - \sin^{-1} a \le \frac{b-a}{\sqrt{1-b^2}}$ for $0<a<b<1$. **[12 Marks]**

98. Show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy = \frac{\pi}{4}$ **[12 Marks]**

99. Let $f(x)=\begin{cases} x^p \sin\frac{1}{x}, x\ne 0 \\ 0 \qquad x=0 \end{cases}$. Obtain condition on p such that (i)f is continuous at $x=0$ and (ii) f is differentiable at $x=0$ **[15 Marks]**

100. Consider the set of triangles having a given base and a given vertex angle show that the triangle having the maximum area will be isosceles **[15 Marks]**

101. If the roots of the equation $(\lambda-u)^3+(\lambda-v)^3+(\lambda-w)^3=0$ in λ are x,y,z. show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=-\frac{2(u-v)(v-w)(w-u)}{(x-y)(y-z)(z-x)}$. **[15 Marks]**

102. Find the center of gravity of the region bounded by the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$ and both axes I the first quadrant the density being $\rho=kxy$ where k is constant. **[15 Marks]**

2001

103. Let be defined on by setting $f(x)=x$ if x is rational and $f(x)=1-x$ if x is irrational show that is continuous at $x=\frac{1}{2}$ but is discontinuous at every other point. **[12Marks]**

104. Test the convergence of $\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}}dx$. **[12 Marks]**

105. Find the equation of the cubic curve which has the same asymptotes as $2x(y-3)^2=3y(x-1)^2$ and which touches the axis at the origin and passes though the point $(1,1)$. **[15 Marks]**

106. Find the maximum and minimum radii vectors of the section of the surface $(x^2+y^2+z^2)=a^2x^2+b^2y^2+c^2z^2$ by the plane $lx+my+nz=0$ **[15 Marks]**

107. Evaluate $\iiint (x+y+z+1)^2 dxdydz$ over the region defined by $x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1$

[15 Marks]

108. Find the volume of the solid generated by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line

[15 Marks]

2000

109. Use the mean value theorem to prove that $\frac{2}{7} < \log 1.4 < \frac{2}{5}$.

[12 Marks]

110. Show that $\iint x^{2l-1} y^{2m-1} dxdy = \frac{1}{4} r^{2(l+m)} \frac{\Gamma l \, \Gamma m}{\Gamma(l+m+1)}$ for all positive values of and laying the circle $x^2 + y^2 = r^2$.

[12 Marks]

111. Find the center of gravity of the positive octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ if the density varies as xyz

[15 Marks]

112. Let $f(x) = \begin{cases} 2, & x \text{ is irrational} \\ 1, & x \text{ is rational} \end{cases}$ show that if is not Riemann integrable on $[a,b]$ **[15 Marks**

113. Show that $\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = (-1)^n \frac{n!}{x^{n+1}}\left(\log x - 1 - \frac{1}{2} - \frac{1}{3} .... - \frac{1}{n}\right)$ **[15 Marks]**

114. Find constant a and b for which $F(a,b) = \int_0^{\pi} \{\log x - ax^2 + bx^2\} dx$ is a minimum **[15 Marks]**

1999

115. Determine the set of all points where the function $f(x) = \frac{x}{1+|x|}$ is differentiable. **[20 Marks]**

116. Find three asymptotes of the curve $x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y - 10 = 0$. Also find the intercept of one asymptote between the other two. **[20 Marks]**

117. Find the dimensions of a right circular cone of minimum volume which can be circumscribed about a sphere of radius a. **[20 Marks]**

118. If f is Riemann integral over every interval of finite length and $f(x+y) = f(x) + f(y)$ for every pair of real numbers x and y show that $f(x) = cx$ where $c = f(1)$ **[20 Marks]**

119. Show that the area bounded by cissoids $x = a\sin^2 t, y = a\frac{\sin^3 t}{\cos t}$ and its asymptote is $\frac{3\pi a^2}{4}$

[20 Marks]

120. Show that $\iint x^{m-1} y^{n-1}$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^m b^n}{4} \frac{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{m}{2} + \frac{n}{2} + 1\right)}$

[20 Marks]

1998

121. Find the asymptotes of the curve $(2x-3y+1)^2(x+y)-8x+2y-9=0$ and show that they intersect the curve again in their points which lie on a straight line. **[20 Marks]**

122. A thin closed rectangular box is to have one edge n times the length of another edge and the volume of the box is given to be v . Prove that the least surface s is given by $ns^3 = 54(n+1)^2 v^2$ **[20 Marks]**

123. If $x+y=1$, Prove that $\frac{d^n}{dx^n}(x^n y^n) = n!\left[y^n - \binom{n}{1}^2 y^{n-1}x + \binom{n}{2}^2 y^{n-2}x^2 + \ldots + (-1)^n x^n\right]$ **[20 Marks]**

124. Show that $\int_0^\infty \frac{x^{p-1}}{(1+x)^{p+q}}dx = B(p,q)$ **[20 Marks]**

125. Show that $\iiint \frac{dxdydz}{\sqrt{(1-x^2-y^2-z^2)}} = \frac{\pi^2}{8}$ Integral being extended over all positive values of x,y,z for which the expression is real. **[20 Marks]**

126. The ellipse $b^2x^2+a^2y^2=a^2b^2$ is divided into two parts by the line $x=\frac{1}{2}a,$ and the smaller part is rotated through for right angles about this line. Prove that the volume generated is $\pi a^2 b\left\{\frac{3\sqrt{3}}{4}-\frac{\pi}{3}\right\}$ **[20 Marks]**

1997

127. Suppose $f(x) = 17x^{12} - 124x^9 + 16x^3 - 129x^2 + x - 1$ determine $\frac{d}{dx}(f^{-1})$ if $x=-1$ it exists. **[20 Marks]**

128. Prove that the volume of the greatest parallelepiped that can be inscribe in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1\,\frac{8abc}{3\sqrt{3}}$ **[20 Marks]**

129. Show that the asymptotes of the cut the curve $(x^2-y^2)(y^2-4x^2)+6x^3-5x^2y-3xy^2+zy^3-x^2+3xy-1=0$ again in eight points which lie on a circle of radius 1. **[20 Marks]**

130. An area bounded by a quadrant of a circle of radius a and the tangent at its extremities revolve about one of the tangents. Find the volume so generated. **[20 Marks]**

131. Show how the changes of order in the integral $\int_0^\infty\int_0^\infty e^{-xy}\sin x\; dxdy$ leads to the evaluation of $\int_0^\infty \frac{\sin x}{x}dx$ hence evaluate it. **[20 Marks]**

132. Show that in $\Gamma n\,\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\Gamma 2n$ where $n>0$ and Γn denote gamma function. **[20 Marks]**

1996

133. Find the asymptotes of all curves $4(x^4+y^4)-17x^2y^2-4x(4y^2-x^2)+2(x^2-2)=0$ and show that they pass thought the point of intersection of the curve with the ellipse $x^2+4y^2=4$. **[20 Marks]**

134. Show that any continuous function defined for all real x and satisfying the equation $f(x)=f(2x+1)$ for all x must be a constant function. **[20 Marks]**

135. Show that the maximum and minimum of the radii vectors of the section of the surface $(x^2+y^2+z^2)^2 = \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$ by the plane $\lambda x + \mu y + \upsilon z = 0$ are given by the equation

$\frac{a^2\lambda^2}{1-a^2r^2}+\frac{b^2\mu^2}{1-b^2r^2}+\frac{a^2v^2}{1-c^2r^2}=0$. **[20 Marks]**

136. If $u = f\left(\frac{x}{a},\frac{y}{b},\frac{z}{c}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$ **[20 Marks]**

137. Evaluate $\int_0^\infty\int_0^\infty \frac{e^{-y}}{y}dxdy$. **[20 Marks]**

138. The area cut off from the parabola $y^2 = 4ax$ by chord joining the vertex to an end of the latus rectum is rotated though four right angles about the chord. Find the volume of the solid so formed. **[20 Marks]**

1995

139. If g is the inverse of f and $f'(x) = \frac{1}{1+x^3}$ prove that $g(x) = 1+[g(x)]^3$ **[20 Marks]**

140. Taking the nth derivative of $(x^n)^2$ in two different ways show that $1+\frac{n^2}{1^2}+\frac{n^2}{1^2.2^2}+\frac{n^2(n-1)^2}{1^2.2^2.3^2}+\ldots$ to $(n+1)term = \frac{(2n)!}{(n!)^2}$ **[20 Marks]**

141. Let $f(x,y)$ which possesses continuous partial derivatives of second order be a homogeneous function of x and y off degree n prove that $x^2f_{xx}+2xyf_{xy}+y^2f_{yy}=n(n-1)f$. **[20 Marks]**

142. Find the area bounded by the curve $\left(\frac{x^2}{4}+\frac{y^2}{9}\right)=\frac{x^2}{4}-\frac{y^2}{9}$. **[20 Marks]**

143. Let $f(x),\ x\geq 1$ be such that the area bounded by the curve $y = f(x)$ and the lines $x=1, x=b$ is equal to $\sqrt{1+b^2}-\sqrt{2}$ for all $b\geq 1$. Does f attain its minimum? If so, what is its values? **[20 Marks]**

144. Show that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)\ldots\Gamma\left(\frac{n-1}{n}\right)=\frac{(2\pi)}{\sqrt{n}}\frac{n-1}{2}$. **[20 Marks]**

1994

145. $f(x)$ Is defined as follows: $f(x) = \begin{cases} \frac{1}{2}(b^2-a^2) & \text{if } 0<x\leq a \\ \frac{1}{2}b^2-\frac{x^2}{6}-\frac{a^2}{3x} & \text{if } a<x\leq b \\ \frac{1}{3}\left(\frac{b^3-a^3}{x}\right) & \text{if } x>b \end{cases}$. Prove that $f(x)$ and $f'(x)$ are continuous but $f''(x)$ is discontinuous. **[20 Marks]**

146. If α and β lie between the least and greatest values of a,b,c prove that

$$\begin{vmatrix} f(a) & f(b) & f(c) \\ \phi(a) & \phi(b) & \phi(c) \\ \psi(x) & \psi(b) & \psi(c) \end{vmatrix} = K \begin{vmatrix} f(a) & f'(\alpha) & f(\beta) \\ \phi(a) & \phi'(\alpha) & \phi(\beta) \\ \psi(x) & \psi'(\alpha) & \psi(\beta) \end{vmatrix}$$ where $K = \frac{1}{2}(b-c)(c-a)(a-b)$ **[20 Marks]**

147. Prove that all rectangular parallelepipeds of same volume, the cube has the least surface **[20 Marks]**

148. Show that means of beta function that $\int_t^z \frac{dx}{(z-x)^{1-\alpha}(x-t)^{\alpha}} = \frac{\pi}{\sin \pi\alpha}(0 < \alpha < 1)$. **[20 Marks]**

149. Prove that the value of $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ taken over the volume bounded by the co-ordinate planes and the plane $x+y+z=1$ is $\frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$. **[20 Marks]**

150. The sphere $x^2+y^2+z^2=a^2$ is pierced by the cylinder $(x^2+y^2)^2 = a^2(x^2-y^2)$ prove by the cylinder $(x^2+y^2)^2 = a^2(x^2-y^2)$ is $\frac{8a^3}{3}\left[\frac{\pi}{4}+\frac{5}{3}=\frac{4\sqrt{2}}{3}\right]$ **[20 Marks]**

1993

151. Prove that $f(x) = x^2 \sin\frac{1}{x}, x \neq 0$ and $f(x) = 0x = 0$ for is continuous and differentiable at $x=0$ but its derivative is not continuous there. **[20 Marks]**

152. If $f(x), \phi(x), \psi(x)$ have derivative when $a \le x \le b$ show that there is a value c of x lying between a and b such that $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f(c) & \phi(c) & \psi(c) \end{vmatrix} = 0$ **[20 Marks]**

153. Find the triangle of maximum area which can be inscribed in a circle **[20 Marks]**

154. Prove that $\int_0^\infty e^{-ax^2}dx = \frac{\sqrt{\pi}}{2\sqrt{a}}(a>0)$ deduce that $\int_0^\infty x^{2n}e^{-x^2}dx = \frac{\sqrt{\pi}}{2^{n+1}}[1.3.5...(2n-1)]$ **[20 Marks]**

155. Defined Gamma function and prove that $\Gamma n \, \Gamma\left(n+\frac{1}{2}\right) = \frac{\Gamma \pi}{2^{2n-1}}\Gamma 2n$ **[20 Marks]**

156. Show that volume common to the sphere $x^2+y^2+z^2=a^2$ and the cylinder $x^2+y^2=ax$ is $\frac{2a^2}{9}(3\pi-4)$. **[20 Marks]**

1992

157. If $y = e^{ax}\cos bx$ prove that $y_2 - 2ay_1 + (a^2+b^2)y = 0$ and hence expand $e^{2x}\cos bx$ in powers of x Deduce the expansion of e^{ax} and $\cos bx$. **[20 Marks]**

158. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$ then prove that $dx^2+dy^2+dz^2 = dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2$. **[20 Marks]**

159. Find the dimension of the rectangular parallelepiped inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that has greatest volume. **[20 Marks]**

160. Prove that the volume enclosed by the cylinders $x^2+y^2=2ax, z^2=2$ axis $\frac{128a^3}{15}$ **[20 Marks]**

161. Find the centre of gravity of the volume formed by revolving the area bounded by the parabolas $y^2=4ax$ and $x^2=4by$ about the x-axis **[20 Marks]**

162. Evaluate the following integral in terms of Gamma function $\int_{-1}^{1}(1+x)^p(1-x)^q\,dx, \quad [p>-1, q>-1]$ and prove that $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)=\frac{2}{\sqrt{3}}\pi$ **[20 Marks]**