ME 521 Fall 2007 Professor John M. Cimbala Lecture 39 12/03/2007

Today, we will:

- Briefly discuss Oseen's improvement to Stokes equations
- Briefly discuss compressible Couette flow
- If time, begin discussing laminar boundary layers – the last topic of the semester

Problems with Stokes equations:

1) Not valid for 2-D flows

2) Problems as $r \to \infty$, inertial terms must be there

⋮

We must include inertial effects to improve on Stokes eq's.

Oseen → Add a linearized inertial term

Let $\vec{u} = U\vec{i} + \vec{u}'$ → a perturbation velocity

Stokes: $$0 = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

$$\boxed{\underbrace{\rho U \frac{\partial u_i'}{\partial x_1}}_{\text{linear term}} = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i'}{\partial x_k \partial x_k}}$$ Oseen's eq.

- Allows us to solve 2-D problems
- Removes the problems at $r \to \infty$
- Still linear
- ρ is back! → get dependence on Re

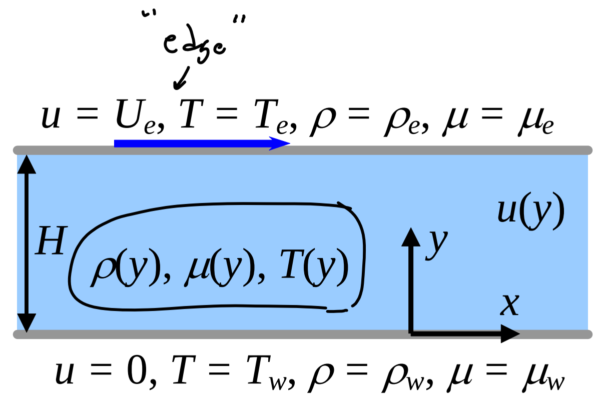
E. Compressible Flows (Laminar)

1. Introduction

So far our flows have cont, mom uncoupled from energy eq. } small ΔT
↓ solve for $\vec{u}$ separately. Then solve for T

If ΔT is large → ρ, μ, etc are now fncts of T ∴ eqs are coupled.

2. Example **Compressible Couette Flow**:



Given: A Newtonian, but *compressible* fluid flows between two infinite parallel flat plates as sketched. The following assumptions/approximations are made:

1. All flow field variables and fluid properties are independent of x (nothing special about any particular x location) $\partial/\partial x(\ ) = 0$
2. The flow is steady: $\partial(\text{anything})/\partial t = 0$
3. The flow is two-dimensional: $\partial(\text{anything})/\partial z = 0$ and $w = 0$
4. The flow is parallel ($v = 0$ everywhere)
5. Neglect gravity
6. There is no imposed pressure gradient ($\partial p/\partial x = 0$), therefore p = constant

(*a*) To do: Calculate the x-component of velocity, $u(y)$ and the temperature $T(y)$.

Solution:

X-mom: Compressible [Cauchy's eq.]

$$\rho \frac{Du_i}{Dt} = \cancel{\rho g_i} + \frac{\partial}{\partial x_j}\tau_{ij}$$

→ compressible constitutive eq. for Newtonian fluid

Expand: Eq. 4.37 → $\tau_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda \frac{\partial u_m}{\partial x_m}\delta_{ij}$

$$\rho\left[\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{steady}} + u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{u=u(y)} + \underbrace{\cancel{v}}_{v=0}\frac{\partial u}{\partial y} + w\underbrace{\cancel{\frac{\partial u}{\partial z}}}_{2\text{-D}}\right] = \frac{\partial}{\partial x}\left[\underbrace{\cancel{-p}}_{\partial p/\partial x=0} + 2\mu\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{u=u(y)} + \lambda\left(\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{u=u(y)} + \underbrace{\cancel{\frac{\partial v}{\partial y}}}_{v=0} + \underbrace{\cancel{\frac{\partial w}{\partial z}}}_{2\text{-D}}\right)\right]$$

$$+ \boxed{\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \underbrace{\cancel{\frac{\partial v}{\partial x}}}_{v=0}\right)\right]} + \underbrace{\cancel{\frac{\partial}{\partial z}}}_{2\text{-D}}[\sim]$$

can use d instead of ∂ since μ & u = fncs of y only reduce to $\boxed{\dfrac{d}{dy}\left(\mu \dfrac{du}{dy}\right) = 0}$. Let $\tau = \mu \dfrac{du}{dy}$

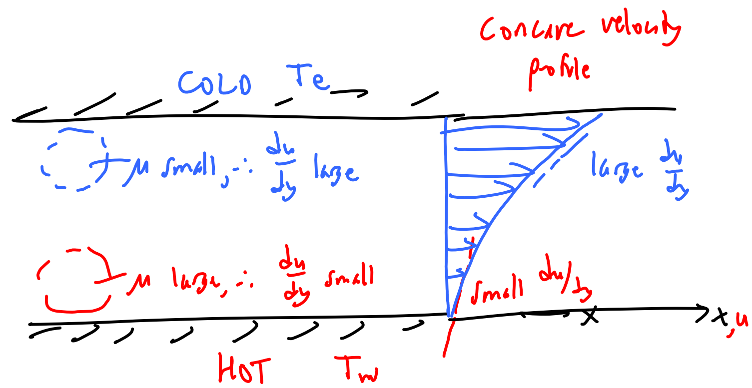
$$\searrow \quad \frac{d\tau}{dy} = 0 \Rightarrow \underline{\underline{\tau = \text{constant}}}$$

C. Qualitative result: $\boxed{\mu \dfrac{du}{dy} = \text{constant}}$

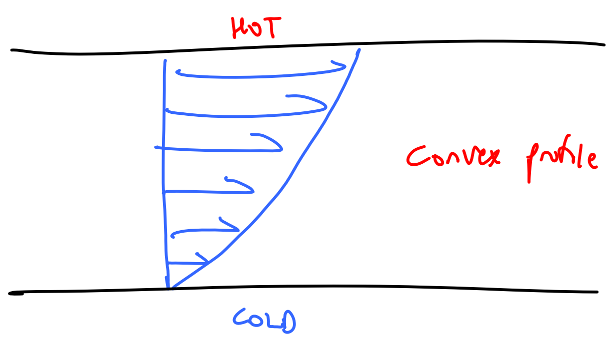
$\mu \neq$ const, but rather $\mu = \mu(T)$ in general, $\because T = T(y) \ni \mu = \mu(y)$

· Gases $\mu \uparrow$ as $T \uparrow$

If $T_w > T_e$

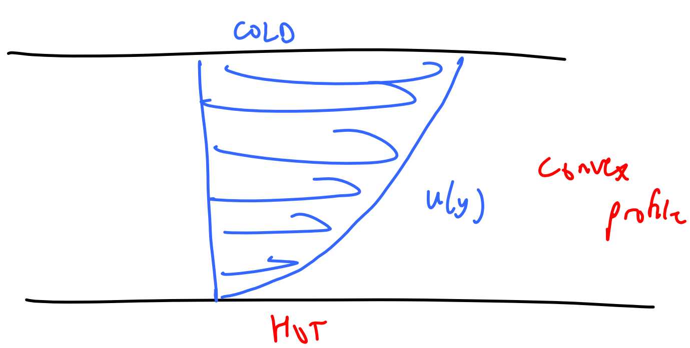


If $T_w < T_e$



· Liquid — most liquids have $\mu \downarrow$ as $T \uparrow$

If $T_w > T_e$



VII LAMINAR BOUNDARY LAYERS

A. Introduction • This is an approximation — solving a reduced or simplified form of N-S eq.

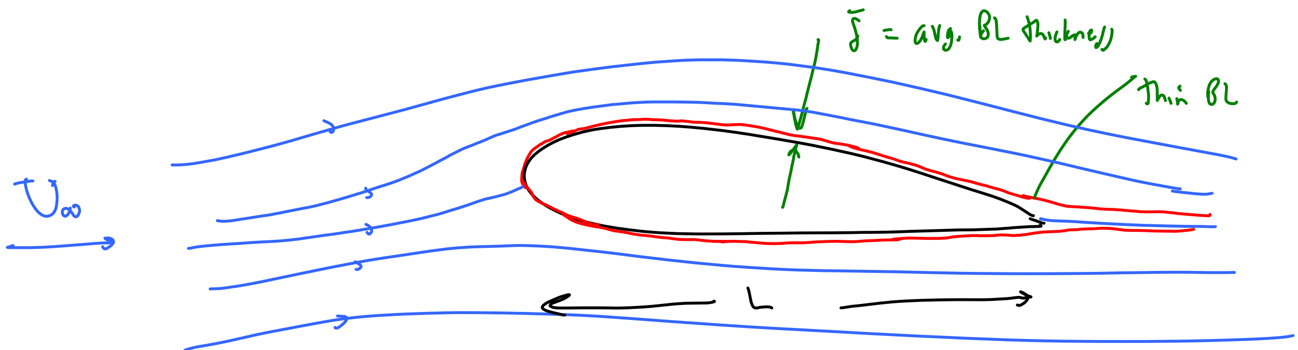
1. Defn: **Laminar BL** is a thin layer in which viscous forces & rotationality are important. ✮

When is this approx. valid?

rate of downstream convection (advection) >> rate of lateral diffusion ✮

(flow is swept downstream much faster than viscous effects can spread laterally)

2. E.g. Flow over an airfoil (2-D)



Time scale → create $\boxed{t \sim \frac{L}{U_\infty}}$ $t \approx$ time for a fluid particle to travel along the body

↓ convective time scale

Order of magnitude

typically magnitude between 0.1 & 10

Viscous diffusion → recall Stokes first problem $\delta \sim \sqrt{\nu t}$ → U

Lets let $\boxed{\bar{\delta} \sim \sqrt{\nu \frac{L}{U_\infty}}}$

3. B.L. Approximation is that $\boxed{\bar{\delta} \ll L} \rightarrow \frac{L}{\bar{\delta}} \gg 1$

$$\frac{L}{\bar{\delta}} \sim \frac{L}{\sqrt{\nu \frac{L}{U_\infty}}} = \sqrt{\frac{L U_\infty}{\nu}} \gg 1$$

Define $Re_L = \frac{U_\infty L}{\nu}$ (Re based on body length)

$$\therefore \boxed{\text{BL approx. is valid when } \sqrt{Re_L} \gg 1}$$

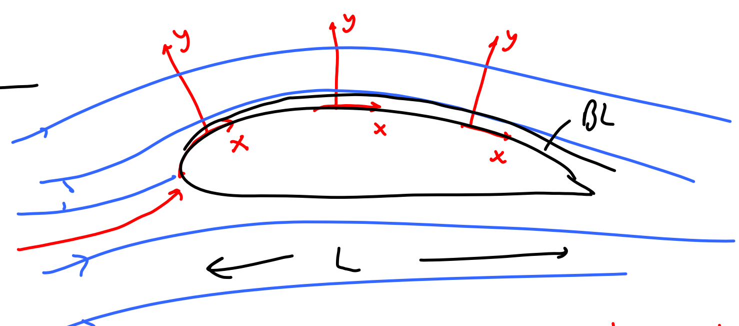
High Reynolds # flows

B. Differential Eqs of motion:

1. Assumptions
   - 2-D flow in x-y plane
   - Incompressible, laminar flow
   - ignore gravity
   - pressure → will look at $p - p_\infty$ instead of p itself (a kind of gage pressure)

2. BL coordinate system

Body-fitted coordinate system



3. Nondimensionalize the eqs & normalizing

lengths: $\boxed{\begin{aligned} x &= x' L \\ y &= y' \bar{\delta} \end{aligned}}$ nondimensional

time: $\boxed{t = t' \frac{L}{U_\infty}}$

$v \ll u$ in a BL

velocity: $\boxed{u = u' U_\infty}$ $\boxed{v = v' V_{scale}}$ must Determine this scale